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REAT: A Regional Economic Analysis Toolbox for R

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Abstract. Methods of regional economic analysis are widely used in regional and urban economics as well as in economic geography. This paper introduces the REAT (Regional Economic Analysis Toolbox) package for the programming environment R, which provides a collection of mathematical regional analysis methods in a user-friendly way. The focus is on the identification of regional inequality, beta and sigma convergence, measurement of agglomerations, point-based measures of clustering and accessibility, as well as regional growth. The theoretical basics of the applications are briefly introduced, while the usage of the most important functions is presented and explained using real data.

1 Introduction

Methods of regional economic analysis (or regional analysis) are used frequently in theory-based, empirical studies from regional and urban economics as well as (quantitative) economic geography. These methods aim at analyzing some of the most important issues in the mentioned research fields, including (but not limited to) the existence and evolution of agglomerations, regional economic growth and regional disparities (Capello, Nijkamp, 2009; Dinc, 2015; Farhauer, Kröll, 2014; Schätzl, 2000). In any of the mentioned fields, a growing amount of quantitative data has to be processed when using traditional or novel methods and models of regional analysis. This paper introduces the package (add-on) `REAT` (Regional Economic Analysis Toolbox) (Wieland, 2019) for the programming environment `R` (R Core Team, 2018a). The package provides a collection of mathematical regional analysis applications, designed in a relatively user-friendly way.

The main topics in the regional analysis context can be summarized as follows, showing also the structure of the present paper with respect to the presented approaches and their application in `REAT`:

1. Identifying regional inequality (or regional disparities) using indicators of concentration and/or dispersion (Section 2)
2. Regional disparities over time leading to the concept of beta and sigma convergence (Section 3)
3. Measuring agglomerations, which means the specialization of regions and the spatial concentration of industries as well as more complex cluster indices (Section 4)
4. Point-based measures of clustering and accessibility (Section 5)
5. Regional growth, especially shift-share analysis (Section 6)

Note that, in its original form, the open source software `R` is a command-line environment including a lot of mathematical and statistical features. For the installation of `R` and its packages as well as the basics of navigation and implemented statistical functions, see the `R` documentations (R Core Team, 2018b). A good supplement for working with `R` is `RStudio` (RStudio Team, 2016). The `REAT` package deals with several `R` data types: The most functions require and calculate `numeric vectors`, but, in some cases, also objects of type `matrix`, `data frame` and `list`, depending on the complexity of calculation. For a quick introduction to the data types in `R` and their properties, see e.g. Kabacoff (2017).

2 Concentration, dispersion and regional disparities

2.1 Indicators of concentration and dispersion

Regional disparities are a frequent topic in economic geography and regional economics. The spatial inequality with respect to e.g. regional output, income or employment is an essential element of polarization theory (Myrdal, 1957) and "New Economic Geography" (Krugman, 1991; Fujita et al., 2001). Assessing regional disparities is possible using concentration and dispersion indicators, which belong to the univariate and descriptive analysis in statistics. Apart from regional economics, these measures are used in several contexts, such as competition economics (market concentration of firms) or welfare economics (income inequality). For a review of the most common indicators with respect to regional inequality, see Portnov, Felsenstein (2010), for studies comparing different indicators in the regional economic context using empirical data, see e.g. Gluschenko (2018); Habánik et al. (2013); Huang, Leung (2009); Palan (2017); Petrakos, Psycharis (2016).

Concentration is operationalized as the discrepancy between an empirical distribution of a variable x (e.g. annual turnover, income, gross domestic product [GDP]) with n observations or objects (e.g. competing firms, households, regions) and a (theoretical) equal distribution or a reference distribution (e.g. population distribution). Dispersion indicators aim at the deviation from the arithmetic mean of x, $\bar{x}$. In this context, Portnov, Felsenstein (2005, 2010) distinguish between measures of deprivation and variation.

Typical measures of regional disparities are the Gini coefficient, the Herfindahl-Hirschman index and the coefficient of variation (Lessmann, 2005). The most popular measure of concentration is the Gini coefficient (Gini, 1912) in combination with the Lorenz curve (Lorenz, 1905). There are several calculation approaches for the Gini coefficient, all producing the same result. The Lorenz curve is a graphical indicator, showing the deviation of the empirical shares of the regarded variable x from a (theoretical) equal distribution. Another well-known indicator is the Herfindahl-Hirschman index, which was developed independently by Hirschman (1945) and Herfindahl (1950), both in the context of competition economics. Several other concentration indicators are also applied in the fields of regional economics with respect to regional disparities, such as the Hoover coefficient (Hoover, 1936) and the Theil coefficient (Theil, 1967).

Except for the standard deviation, whose unit is equal to the unit of x, all common indicators are dimensionless. Most of them (except for standard deviation and coefficient of variation) have a fixed value range, normally between zero (indicating complete equality/dispersion) and one (indicating complete inequality/concentration).

Most of the common indicators are mathematically formulated in an unweighted and in a weighted form, while, in the context of regional disparities, the latter is mostly done using the regions' proportion of the total (e.g. national) population (Doran, Jordan 2013; Lessmann 2014; Mussini 2017; Petrakos, Psycharis 2016; for a critical discussion of weighting these coefficients, see Gluschenko 2018). In the literature, there are different formulations where the weighted coefficients also include a weighted arithmetic mean. Note that, in the case of the population-weighted Gini coefficient, a weighted arithmetic mean is mandatory to keep the indicators' value range.

Especially when dealing with GDP per capita as an indicator of regional economic output, several recent studies use dispersion measures rather than concentration measures, especially the (weighted) coefficient of variation (e.g. Lessmann 2005, 2014, 2016;

Lessmann, Seidel 2017; Petrakos, Psycharis 2016). This dispersion indicator is a dimensionless normalization of the standard deviation. Weighting the coefficient of variation with population shares was introduced by Williamson (1965), which has led to calling this coefficient the Williamson index. As regional incomes or outputs are not normally distributed in most cases, resulting in biased arithmetic means used in the calculation of dispersion measures, the regarded variable may be log-transformed, which means replacing x_i with $\log(x_i)$ in the calculations.

Table 1 shows the common indicators, including their (population-)weighted and their normalized form (if there exist any) and the corresponding value ranges. The formulae are shown in a way that includes several ways of application. The regarded variable is always named x_i, while the (population) weighting is called w_i. Some indicators, such as the Hoover or the Coulter coefficient, require a variable representing a reference distribution the shares of x_i are compared to. This reference is *not* a weighting. However, in many studies, the regional population is also used for the reference distribution. In these cases, reference and weighting are the same data. The reference distribution may also be equal to $1/n$.

Several indicators are also used for the analysis of regional specialization or the spatial concentration of industries, such as the Hoover coefficient or the Herfindahl-Hirschman index or its inverse ($1/HHI$; also known as the "equivalent number" in the competition context). Other coefficients of concentration and specialization are discussed in Section 4. The last coefficient in Table 1, the mean square successive difference (von Neumann et al., 1941) is a measure for time variability not originating from but also transferable to regional economics.

2.2 Application in REAT

2.2.1 REAT functions for concentration and dispersion indicators

Table 2 shows the functions for concentration and dispersion measures implemented in the `REAT` package. All functions require at least one argument, a `numeric vector` with a length equal to n, containing the regarded variable x (e.g. income) with i observations (e.g. regions), where $i = 1, ..., n$. This data may be a single `vector` or a column of a `data frame` or `matrix`.

An optional weighting of the vector `x` can be done using the function argument `weighting` which is also a `numeric vector` of length n. By default, the functions remove missing (`NA`) values. The `hoover()` function always needs a reference distribution (see the Hoover coefficient formula in Table 1), which is stated via the `ref` argument, also requiring a `numeric vector` of length n. If no reference variable is stated (`ref = NULL`), the reference is set to $1/n$.

All functions (except for `disp()`) return the single value of the computed coefficient. In the relevant cases (`gini()`, `gini2()`, `herf()` and `cv()`), a normalization of the coefficient is possible using the function argument `coefnorm = TRUE`, returning the normalized coefficient instead of the raw coefficient. The function `disp()` is a wrapper for all mentioned functions, calculating all coefficients (except for the *MSSD*) at once for one vector `x` or a set of variables/columns from a `data frame` or `matrix`.

Note that there are two functions for the Gini coefficient, `gini()` and `gini2()`, both producing the same result in the unweighted case. The former function is designed for income inequality, where the `weighting` option is designed for the calculation of the Gini coefficient for groups (e.g. income classes), where the weighting represents the group mean. The function `gini2()` is designed for the population-weighted analysis of regional inequality.

2.2.2 Application example: Small-scale regional disparities in health care provision

Regional inequality with respect to health care providers is a topic of high societal significance. In Germany, the health care planning system (*Kassenärztliche Bedarfsplanung*) attempts to flatten the disparities of local health care provision (Kassenärztliche Bundesvereinigung, 2013). Here, we analyze small-scale regional disparities in health care

Table 1: Indicators of concentration and dispersion for analyzing regional disparities

Indicator	Unweighted	Weighted	Normalized
Gini	$G = \frac{1}{2n^2\bar{x}} \sum_{i=1}^{n} \sum_{j=1}^{n} \lvert x_i - x_j \rvert$ $0 \leq G \leq 1 - \frac{1}{n}$	$G^w = \frac{1}{2\bar{x}^w} \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \lvert x_i - x_j \rvert$ $0 \leq G \leq 1 - \frac{1}{n}$	$G^* = \frac{n}{n-1} G$ $0 \leq G^* \leq 1$
HHI	$HHI = \sum_{i=1}^{n} (\frac{x_i}{\sum_{i=1}^{n} x_i})^2$ $\frac{1}{n} \leq HHI \leq 1$		$HHI^* = \frac{HHI - \frac{1}{n}}{1 - \frac{1}{n}}$ $0 \leq HHI^* \leq 1$
Hoover	$HC = \frac{1}{2} [\sum_{i=1}^{n} \lvert \frac{x_i}{\sum_{i=1}^{n} x_i} - \frac{r_i}{\sum_{i=1}^{n} r_i} \rvert]$ $0 \leq HC \leq 1$	$HC^w = \frac{1}{2} [\sum_{i=1}^{n} w_i \lvert \frac{x_i}{\sum_{i=1}^{n} x_i} - \frac{r_i}{\sum_{i=1}^{n} r_i} \rvert]$ $0 \leq HC \leq 1$	
Theil	$TC = \frac{1}{n} \sum_{i=1}^{n} \ln(\frac{\bar{x}}{x_i})$ $0 \leq TC \leq 1$	$TC^w = \frac{1}{n} \sum_{i=1}^{n} w_i \ln(\frac{\bar{x}}{x_i})$ $0 \leq TC^w \leq 1$	
Coulter		$CC = \sqrt{\frac{1}{2} [\sum_{i=1}^{n} w_i (\frac{x_i}{\sum_{i=1}^{n} x_i} - \frac{r_i}{\sum_{i=1}^{n} r_i})^2]}$ $0 \leq CC \leq 1$	
Atkinson	$AI = 1 - [\frac{1}{n} \sum_{i=1}^{n} x_i^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$ $0 \leq AI \leq 1$		
Dalton	$\delta = \frac{\log(\frac{1}{n} \sum_{i=1}^{n} x_i)}{\log(\sqrt[n]{\pi_{i=1}^{n} x_i})}$ $0 \leq \delta \leq \infty$		
SD	$s = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$ $0 \leq s \leq \infty$	$s^w = \sqrt{\frac{1}{n} \sum_{i=1}^{n} w_i (x_i - \bar{x})^2}$ $0 \leq s \leq \infty$	see CV
CV	$v = \frac{1}{\lvert \bar{x} \rvert} \sqrt{\frac{1}{n} \sum_{i=1}^{n} (x_i - \bar{x})^2}$ $0 \leq v \leq \infty$	see Williamson	$v^* = \frac{v}{\sqrt{n}}$ $0 \leq v^* \leq 1$
Williamson		$WI = \frac{1}{\lvert \bar{x} \rvert} \sqrt{\frac{1}{n} \sum_{i=1}^{n} w_i (x_i - \bar{x})^2}$ $0 \leq v \leq \infty$	
MSSD	$MSSD = \frac{\sum_{t=1}^{T-1} (x_{t+1} - x_t)^2}{T-1}$		

Notes: x_i is the i-th observation of the regarded variable x (e.g. GDP [per capita] in region i), x_j is the value of the same variable with respect to object j, r_i is the i-th observation of a reference variable (e.g. population), n is the number of objects (e.g. regions), $\bar{x}$ is the arithmetic mean of x: $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$, $\bar{x}^w$ is the weighted arithmetic mean of x: $\bar{x}^w = \frac{1}{n} \sum_{i=1}^{n} w_i x_i$, w_i and w_j are the population weightings: $P_i / \sum_{i=1}^{n} P_i$ and $P_j / \sum_{j=1}^{n} P_j$, where P_i and P_j are the population sizes of regions i and j, respectively, ϵ is an inequality aversion parameter ($0 < \epsilon < \infty$) for the Atkinson index, t is a given time period and T is the number all regarded time periods.
Compiled from: Charles-Coll (2011); Cracau, Durán Lima (2016); Damgaard, Weiner (2000); Gluschenko (2018); Heinemann (2008); Kohn, Öztürk (2013); Portnov, Felsenstein (2005, 2010); Taylor, Cihon (2004); Schätzl (2000); Störmann (2009)

provision in two neighboring German counties (Göttingen and Northeim) using the data on medical practices and local population from Wieland, Dittrich (2016). The data is stored in the datasets `GoettingenHealth1` and `GoettingenHealth2`, both included as example datasets in the `REAT` package. The study area is segmented into 420 districts, representing either city districts of larger cities or villages and hamlets.

The dataset `GoettingenHealth2` contains these 420 regions with an individual ID

Table 2: REAT functions for concentration and dispersion indicators

Indicator	REAT function	Mandatory arguments	Optional arguments	Output
Gini/ Lorenz	gini()	vector x	weighting vector, remove NAs, Lorenz curve, normalization	value: G or G^* or G^w, optional: plot (LC)
	gini2()	vector x	weighting vector P_i, remove NAs, normalization	value: G or G^* or G^w,
	lorenz()	vector x	weighting vector, remove NAs,	plot LC, value: G or G^w and/or G^*
HHI	herf()	vector x	remove NAs, normalization	value: HHI or HHI^* or N_{HHI}
Hoover	hoover()	vector x reference vector r_i	weighting vector P_i, remove NAs	value: HC or HC^w
Theil	theil()	vector x	weighting vector P_i, remove NAs	value: TC or TC^w
Coulter	coulter()	vector x	weighting vector P_i, remove NAs	value: CC
Atkinson	atkinson()	vector x	remove NAs, epsilon	value: AI
Dalton	dalton()	vector x	remove NAs	value: δ
SD	sd2()	vector x	weighting vector, remove NAs, treating as sample	value: s or s^W
CV	cv()	vector x	weighting vector, remove NAs, normalization, treating as sample	value: v or v^W or v^*
Williamson	williamson()	vector x, weighting vector P_i	remove NAs	value: WI
MSSD	mssd()	vector x	remove NAs	value: $MSSD$
All indicators	disp()	vector x or vectors $x_1, x_2, ...$ from dataframe	weighting vector P_i, remove NAs	matrix with 13 (no weighting) or 19 indicators (incl. weighted)

Source: own compilation.

(column `district`) and geographic coordinates (columns `lat` and `lon`, respectively) and the number of general practitioners, psychotherapists and pharmacies located there (columns `phys_gen`, `psych` and `pharm`, respectively) as well as the local population (column `pop`). First, we load the dataset:

```
data(GoettingenHealth2)
```

Now, we investigate how the health care providers are dispersed over the whole area. In the first step, we calculate the Gini coefficient for the concentration of general practitioners using the `REAT` function `gini()`:

```
gini (GoettingenHealth2$phys_gen)
[1] 0.8386269
```

The empirical Gini coefficient is equal to 0.839, indicating a relatively strong concentration. If we want to calculate the normalized (unbiased) indicator instead, we use the same function with the optional argument `coefnorm = TRUE`:

```
gini (GoettingenHealth2$phys_gen, coefnorm = TRUE)
[1] 0.8406284
```

In the same way, we calculate e.g. the Herfindahl-Hirschman index, non-normalized and normalized:

```
herf (GoettingenHealth2$phys_gen)
[1] 0.01528053

herf (GoettingenHealth2$phys_gen, coefnorm = TRUE)
[1] 0.01293036
```

Remember that the minimum of HHI is $1/n$ (here: $1/420 \approx 0.00238$) and the minimum of HHI^* is equal to zero.

If we want to inspect the concentration graphically, we could use the Lorenz curve, which can be plotted using either the functions `gini()` or `lorenz()`. Here, we use `gini()`, tell the function to plot the curve (`lc = TRUE`), and include several graphical parameters (such as `lc.col` for the color of the Lorenz curve or `lcx` and `lcy` for the x/y axes labels). As we want to compare the population distribution to the location distribution, we start by plotting the Lorenz curve for the local population:

```
gini(GoettingenHealth2$pop, lc = TRUE, lsize = 1, le.col = "black",
lc.col = "orange", lcx = "Shares of districts", lcy = "Shares of
providers", lctitle = "Spatial concentration of health care
providers", lcg = TRUE, lcgn = TRUE, lcg.caption =
"Population 2016:", lcg.lab.x = 0, lcg.lab.y = 1)
# Gini coefficient and Lorenz curve for the no. of inhabitants
[1] 0.5840336
```

Now, we overlay the Lorenz curves of general practitioners and psychotherapists, which means adding two more curves (function argument `add.lc = TRUE`):

```
gini(GoettingenHealth2$phys_gen, lc = TRUE, lsize = 1, add.lc = TRUE,
lc.col = "red", lcg = TRUE, lcgn = TRUE, lcg.caption =
"Physicians 2016:", lcg.lab.x = 0, lcg.lab.y = 0.85)
# Adding Gini coefficient and Lorenz curve for the general practitioners
[1] 0.8386269

gini(GoettingenHealth2$psych, lsize = 1, lc = TRUE, add.lc = TRUE,
lc.col = "blue", lcg = TRUE, lcgn = TRUE, lcg.caption =
"Psychotherapists 2016:", lcg.lab.x = 0, lcg.lab.y = 0.7)
# Adding Gini coefficient and Lorenz curve for psychotherapists
[1] 0.9329298
```

Our commands result in the output of Figure 1, showing three Lorenz curves (population, general practitioners and psychotherapists) and the line of equality (diagonal). All three empirical distributions differ from an equal distribution. In about 72% of the regions, representing about 23% of the whole population (orange curve; $G \approx 0.584$), no general practitioner is located (red curve; $G \approx 0.839$). But the psychotherapists are more concentrated, as they are located only in about 13% of all districts (blue curve; $G \approx 0.933$). As we can see, the physicians are more concentrated than the inhabitants but the psychotherapists are more concentrated than the physicians.

Now, we calculate all mentionened concentration and dispersion coefficients at once for all three types of providers using the function `disp()`, including a population weighting:

```
disp(GoettingenHealth2[c(5,6,7)], weighting = GoettingenHealth2$pop)
# column 5 = general practitioners, column 6 = psychotherapists,
# column 7 = pharmacies, column "pop" = local population
```

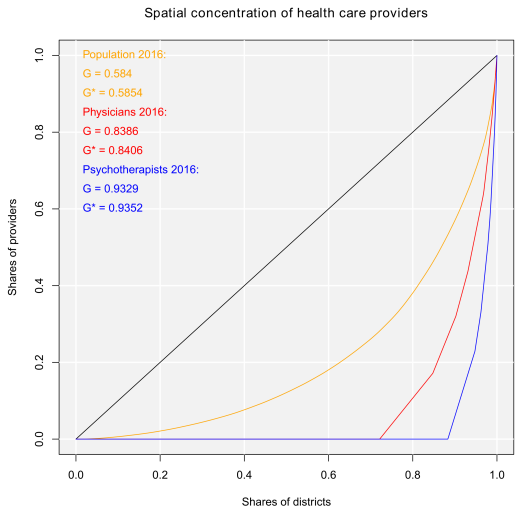


Figure 1: Lorenz curves for the spatial concentration of health care providers

Our output is:

```
Concentration and dispersion measures
Note: w = weighted, n = normalized, eq = equivalent number

                phys_gen         psych         pharm
Gini        0.838626907  0.932929782  0.891547619
Gini n      0.840628403  0.935156345  0.893675418
Gini w      0.629454516  0.770895945  0.705628058
Gini w n    0.630956794  0.772735792  0.707312135
HHI         0.015280527  0.038494685  0.024166667
HHI n       0.012930361  0.036199923  0.021837709
HHI eq     65.442769020 25.977611940 41.379310345
Hoover      0.721428571  0.883333333  0.838095238
Hoover w    0.001852337  0.003130602  0.003418787
Theil                NA           NA           NA
Theil w              NA           NA           NA
Coulter     0.049850824  0.123305927  0.065569205
Atkinson    0.761164110  0.900755425  0.854223763
Dalton               NA           NA           NA
SD          1.714506606  1.095496987  0.865286915
SD w        4.010246439  1.847716870  2.401476794
CV          2.330397328  3.899226565  3.028504203
CV n        0.113847359  0.190489683  0.147952112
Williamson  1.429449565  1.965446423  1.709288672
```

We conclude that any concentration/dispersion measure is the highest for psychotherapists and the lowest for the general practitioners, while the values for pharmacies lie between them. The regional disparities with respect to pharmacies are higher than those with respect to general practitioners, while the most unequal distribution is that of psychotherapists. In other words: The pharmacies are more spatially concentrated than the general practitioners and the psychotherapists are the most concentrated health locations here.

In most cases, population weighting reduces the coefficient values. That is, because districts with a large (small) population have a high (low) impact on the resulting coefficient and the districts without health service providers are also small districts. Furthermore, as the regarded variables contain zero values (which means no health service locations), the Theil coefficient (including the term $\ln(\bar{x}/x_i)$) and the Dalton coefficient (including the n-th root) cannot be computed, resulting in an output of `NA`.

The visible output of any function presented above can be saved in a new `R` object:

```
gini_phys <- gini (GoettingenHealth2$phys_gen)
# save as gini_phys (numeric vector of length = 1)
```

We can simply access our result:

```
gini_phys
[1] 0.8386269
```

The function `disp()` returns a `matrix` with 13 rows (when only unweighted coefficients are computed) or 19 rows (in the case of additional weighted coefficients) and one column for each regarded variable:

```
disp_Goettingen <- disp(GoettingenHealth2[c(5,6,7)],
weighting = GoettingenHealth2$pop)
# save as disp_Goettingen (matrix)
```

We call our results:

```
disp_Goettingen

                phys_gen       psych       pharm
Gini          0.83862691  0.93292978  0.89154762
Gini n        0.84062840  0.93515634  0.89367542
...
```

3 Regional convergence

3.1 The concept of beta and sigma convergence

Regional convergence is derived from (regional) growth theory (for an extensive survey, see Barro, Sala-i Martin 2004) and means the decline of regional disparities *over time*. The neoclassical growth model states that a region's economic output (e.g. GDP per capita) depends on its stock of factors of production, capital and labor (aggregate production function), on condition of constant returns to scale and diminishing marginal product of the factor inputs. As a consequence, regions with a high (low) initial level of factor input grow slower (faster) than "poor" ("rich") regions, what is called beta convergence. It is assumed that all regions converge to the same regional output level (steady-state). Sigma convergence means the decline of regional inequality with respect to regional output over time itself (Allington, McCombie, 2007; Capello, Nijkamp, 2009).

Both types of convergence can be tested empirically, as presented in Table 3. When testing for beta convergence, the natural logarithms of output growth over T time periods in i regions is regressed against the natural logarithms of the initial output values at time t. The original convergence formula was presented by Barro, Sala-i Martin (2004) using a nonlinear least squares (NLS) estimation approach. But in many cases, a linear transformation is used which allows for ordinary least squares (OLS) estimation (Allington, McCombie, 2007; Dapena et al., 2016; Schmidt, 1997; Young et al., 2008). The outcome variable of the convergence equation can be the regional growth between two years (e.g. Young et al. 2008) or the average growth rate per year (e.g. Goecke, Hüther 2016; Puente 2017; Weddige-Haaf, Kool 2017). Significance tests are carried out with t-tests for the regression coefficients and, in the OLS case, the F-test for the significance of R^2.

The estimated parameter of interest is the slope of the model, here denoted β (that is why the modeled process is called *beta* convergence): If $\beta < 0$ and statistically significant, there is *absolute* beta convergence. If additional variables (conditional variables) are included into the convergence equation, we have a test for *conditional* beta convergence. A further interpretation of the β coefficient is possible using the speed of convergence, λ, and H, the so-called half-life, which means the time (measured in the regarded time periods) to reduce the regional disparities by one half (Allington, McCombie, 2007; Schmidt, 1997).

Sigma convergence (which is named after the Greek letter for the standard deviation, σ) can be tested in two ways depending on the number of time periods: The regional

Table 3: Beta and sigma convergence

Type of convergence	Two time periods	More than two time periods
Beta convergence	absolute	
and estimation type	NLS	NLS
	$\frac{1}{T}\ln(\frac{Y_{i,t2}}{Y_{i,t1}}) = \alpha - [\frac{(1-e^{-\beta T})}{T}]\ln(Y_{i,t1}) + \epsilon$	$\frac{1}{T}\sum_{t=1}^{T}\ln(\frac{Y_{i,t+1}}{Y_{i,t}}) = \alpha - [\frac{(1-e^{-\beta T})}{T}]\ln(Y_{i,t1}) + \epsilon$
	OLS	OLS
	$\frac{1}{T}\ln(\frac{Y_{i,t2}}{Y_{i,t1}}) = \alpha + \beta\ln(Y_{i,t1}) + \epsilon$	$\frac{1}{T}\sum_{t=1}^{T}\ln(\frac{Y_{i,t+1}}{Y_{i,t}}) = \alpha + \beta\ln(Y_{i,t1}) + \epsilon$
	conditional	
	NLS	NLS
	$\frac{1}{T}\ln(\frac{Y_{i,t2}}{Y_{i,t1}}) = \alpha - [\frac{(1-e^{-\beta T})}{T}]\ln(Y_{i,t1}) + \theta X_i + \epsilon$	$\frac{1}{T}\sum_{t=1}^{T}\ln(\frac{Y_{i,t+1}}{Y_{i,t}}) = \alpha - [\frac{(1-e^{-\beta T})}{T}]\ln(Y_{i,t1}) + \theta X_i + \epsilon$
	OLS	OLS
	$\frac{1}{T}\ln(\frac{Y_{i,t2}}{Y_{i,t1}}) = \alpha + \beta\ln(Y_{i,t1}) + \theta X_i + \epsilon$	$\frac{1}{T}\sum_{t=1}^{T}\ln(\frac{Y_{i,t+1}}{Y_{i,t}}) = \alpha + \beta\ln(Y_{i,t1}) + \theta X_i + \epsilon$
	$\beta < 0$	$\beta < 0$
	Convergence speed: $\lambda = \frac{-\ln(1+\beta)}{T}$	
	Half-life: $H = \frac{\ln(2)}{\lambda}$	
Sigma convergence	$\sigma_t = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_{i,t} - \bar{Y}_t)^2}$ or	
	$cv_t = \frac{\sigma_t}{\lvert\bar{Y}_t\rvert}$	
	$\frac{\sigma_{t1}}{\sigma_{t2}} > 1$ or	$\sigma = a + bt + \epsilon$ or
	$\frac{cv_{t1}}{cv_{t2}} > 1$	$cv = a + bt + \epsilon$
	Test statistic: $\frac{\sigma^2_{t1}}{\sigma^2_{t2}}$	$b < 0$

Notes: $Y_{i,t}$ is the regional output (e.g. GDP per capita) of region i at time t, $\bar{Y}_t$ is the arithmetic mean of $Y_{i,t}$ for all regions at time t, T is the number of regarded time periods (e.g. years), X_i is a set of other variables (conditions), σ_t is the standard deviation of the regional output of all regions, cv_t is the corresponding coefficient of variation, α, β, θ, a and b are estimated coefficients, ϵ is an error term and n is the number of regions.
Compiled from: Allington, McCombie (2007); Barro, Sala-i Martin (2004); Furceri (2005); Schmidt (1997)

inequality between all regions at time t is measured using the standard deviation, σ_t, or the coefficient of variation, cv_t, for the GDP per capita in its original or natural-logged form. If only two years are regarded, the quotient of both parameters is computed. If e.g. $\sigma_{t1} > \sigma_{t2}$, the regional inequality has declined from $t1$ to $t2$. A significance test can be applied with a simple ANOVA (analysis of variance), where the test statistic is the quotient of the underlying variances (σ^2) (Furceri, 2005; Schmidt, 1997; Young et al., 2008). Within a time series, the dispersion parameter is regressed (and plotted) against time. If the slope coefficient of time is negative, there is sigma convergence (Goecke, Hüther, 2016; Huang, Leung, 2009; Schmidt, 1997).

3.2 Application in REAT

3.2.1 REAT functions for beta and sigma convergence

Table 4 shows the functions for beta and sigma convergence as implemented in `REAT`. The analysis of beta convergence is provided by the functions `betaconv.ols()` and `betaconv.nls()` for OLS and NLS estimation, respectively. Speed of convergence and

Table 4: REAT functions for beta and sigma convergence

Convergence	REAT function	Mandatory arguments	Optional arguments	Output
Beta convergence	betaconv.ols()	vectors $Y_{i,t1}$ and $Y_{i,t2}, ..., Y_{i,T}$, t_1 and t_T	Conditions, scatterplot	visible: model estimates, invisible: list with model estimates and regression data, optional: plot
	betaconv.nls()	vectors $Y_{i,t1}$ and $Y_{i,t2}, ..., Y_{i,T}$, t_1 and t_T	Conditions, scatterplot	visible: model estimates, invisible: list with model estimates and regression data, optional: plot
	betaconv.speed()	values β and T		matrix with λ and H
Sigma convergence	sigmaconv() (when $T = 2$)	vectors $Y_{i,t1}$ and $Y_{i,t2}$, t_1 and t_T	Sigma measure, log, weighting, normalization	visible: estimates, invisible: matrix with estimates
	sigmaconv.t() (when $T > 2$)	vectors $Y_{i,t1}$ and $Y_{i,t2}, ..., Y_{i,T}$, t_1 and t_T	Sigma measure, log, weighting, normalization, line plot	visible: model estimates, invisible: matrix with model estimates, optional: plot
All at once: Beta and sigma convergence	rca()	vectors $Y_{i,t1}$ and $Y_{i,t2}, ..., Y_{i,T}$, t_1 and t_T	Beta estimation, conditions, scatterplot, sigma measure, log, weighting, line plot	visible: model estimates, invisible: list with model estimates and regression data, optional: plot

Source: own compilation.

half-life can be computed with the function `betaconv.speed()`. The ratio test of sigma convergence for two time periods can be done using the function `sigmaconv()`, while a trend regression over time is implemented into the function `sigmaconv.t()`. Both convergence types can be analyzed at once with the function `rca()`, which is a wrapper for all functions mentioned above.

The functions require (at least) two `numeric vectors`, containing the regarded variable Y (e.g. GDP per capita) for at least two different time periods, e.g. from the same `data frame`. Also the start and end time periods (t_1 and t_T) have to be stated. Optionally, a graphical output can be generated (scatterplot for beta convergence, line plot for sigma convergence with respect to longitudinal data). Furthermore, when analyzing sigma convergence, the user can choose whether Y should be log-transformed or not and/or which sigma measure is computed (variance, standard deviation or coefficient of variation; weighted or non-weighted).

Note that, unlike the functions for regional inequality indicators (Section 2), the `REAT` functions for regional convergence distinguish between a *visible* and an *invisible* output. The latter can be saved as a new `R` object. While the visible output shows the main results, the invisible output goes beyond that: `betaconv.ols()`, `betaconv.nls()` and `rca()` return a `list`, which is the most flexible data type in `R`, because it consists of a non-predetermined number of different data objects. Apart from the model results, e.g. the (transformed) regression data is returned in this invisible output.

3.2.2 Application example: Beta and sigma convergence in Germany on the county level

In this example, we look at regional convergence in Germany. The `REAT` package includes the example dataset `G.counties.gdp` with the GDP (gross domestic product), the population and the GDP per capita for the 402 counties ("Kreise") in Germany 1992 to 2014

(complete data only for 2000-2014). First, we load the dataset:

```
data (G.counties.gdp)
```

In our case, we prevent scientific notation of numbers in R and set a limit of 4 digits:

```
options(scipen = 100, digits = 4)
```

We need the columns named gdppcxxxx, containing the GDP per capita for each year, e.g. G.counties.gdp$gdppc2010 contains the GDP per capita for 2010. In the first step, we test absolute beta convergence comparing the years 2010 and 2014 with OLS estimation using the function betaconv.ols():.

```
betaconv.ols (G.counties.gdp$gdppc2010, 2010, G.counties.gdp$gdppc2014,
2014, print.results = TRUE)
# Two years, no conditions (Absolute beta convergence)
```

The output is:

```
Absolute Beta Convergence
Model coefficients (Estimation method: OLS)
           Estimate Std. Error t value     Pr (>|t|)
Alpha      0.104159   0.018934   5.501 0.00000006743
Beta      -0.007373   0.001848  -3.990 0.00007867475
Lambda     0.001850         NA      NA            NA
Halflife 374.640507         NA      NA            NA
Model summary
          Estimate F value df 1 df 2    Pr (>F)
R-Squared  0.03827   15.92    1  400 0.00007867
```

We see that both regression coefficients, α and β, are statistically significant ($t \approx 5.50$ and -3.99, respectively, both $p < 0.001$) and the linear regression model is significant as a whole ($F \approx 15.92$, $p < 0.001$). The negative sign of β shows that, on average, the higher the initial GDP per capita, the lower its growth, which indicates absolute beta convergence. However, the convergence process is very slow: The speed of convergence, represented by λ, shows a harmonization by 0.185% per year. This implies that the output gap will be reduced by 50% in approximately 375 years.

Now we check sigma convergence for the same time using the function sigmaconv(). We choose the coefficient of variation as measure, while using the GDP per capita values in their original form:

```
sigmaconv (G.counties.gdp$gdppc2010, 2010, G.counties.gdp$gdppc2014,
2014, sigma.measure = "cv", print.results = TRUE)
# Using the coefficient of variation
```

The output is:

```
Sigma convergence for two periods (ANOVA)
         Estimate F value df1 df2 Pr (>F)
CV 2010   0.03416      NA  NA  NA      NA
CV 2014   0.03316      NA  NA  NA      NA
Quotient  1.03004   1.038 401 401  0.7117
```

The coefficient of variation is a little smaller in 2014, which means the spatial inequality declined between 2010 and 2014. The quotient of the variances is slightly above one ($F = \sigma^2_{2010}/\sigma^2_{2014} \approx 1.04$), but not statistically significant ($p \approx 0.71$).

When analyzing regional convergence with REAT, it is preferable (and more convenient) to use the wrapper function rca(). Instead of repeating the results above, we test for (absolute) beta and sigma convergence between 2000 and 2014. The analysis of sigma convergence uses trend regression (function argument sigma.type = "trend") for the coefficient of variation (sigma.measure = "cv"). We also want plots for both convergence types (beta.plot = TRUE and sigma.plot = TRUE, respectively) with specific axis labels (e.g. beta.plotX = "Ln (initial GDP p.c.)"). Our code is:

```
rca (G.counties.gdp$gdppc2000, 2000, G.counties.gdp[55:68], 2014,
conditions = NULL, sigma.type = "trend", sigma.measure = "cv",
beta.plot = TRUE, beta.plotLine = TRUE, beta.plotX =
"Ln (initial GDP p.c.)", beta.plotY = "Ln (av. growth GDP p.c.)",
beta.plotTitle = "Beta convergence of German counties 2000-2014",
sigma.plot = TRUE, sigma.plotY = "cv of ln (GDP p.c.)",
sigma.plotTitle = "Sigma convergence of German counties 2000-2014")
# 14 years: 2000 (column 55) to 2014 (column 68)
# no conditions (Absolute beta convergence)
# with plots for both beta and sigma convergence
```

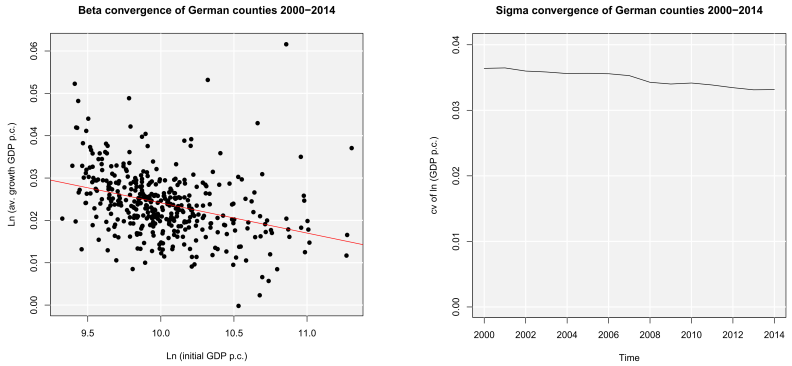
This results in the following output:

```
Regional Beta and Sigma Convergence

Absolute Beta Convergence
Model coefficients (Estimation method: OLS)
             Estimate Std. Error t value                  Pr (>|t|)
Alpha       0.0954564  0.0099087   9.634 0.000000000000000000006845
Beta       -0.0071323  0.0009885  -7.215 0.000000000002719258225550
Lambda      0.0005113         NA      NA                          NA
Halflife 1355.7282963         NA      NA                          NA
Model summary
          Estimate F value df 1 df 2           Pr (>F)
R-Squared   0.1152   52.06    1  400 0.000000000002719

Sigma convergence (Trend regression)
            Estimate Std. Error t value        Pr(>|t|)
Intercept  0.5523659 0.03084855   17.91 0.0000000001526
Time      -0.0002579 0.00001537  -16.78 0.0000000003446
Model summary
          Estimate F value df 1 df 2         Pr (>F)
R-Squared   0.9558   281.4    1   13 0.0000000003446
```

This function also produces the plots in Figures 2a and 2b, both showing a declining curve, which is a first indication of both beta and sigma convergence. The beta convergence model is statistically significant ($F \approx 52.06$, $p < 0.001$), as well as the coefficients α ($t \approx 9.63$, $p < 0.001$) and β ($t \approx -7.21$, $p < 0.001$). Again, we find evidence for absolute beta convergence because of a negative slope ($\beta \approx -0.007$). The trend regression model for sigma convergence is significant ($F \approx 281.4$, $p < 0.001$). The slope is significant and negative ($b \approx -0.00026$, $t \approx 17.91$, $p < 0.001$), which indicates sigma convergence. However, both types of convergence can be regarded as very slow processes: The half-life value shows that, resulting from the beta convergence model, the regional disparities in GDP per capita will be halved in approximately 1,356 years. When looking at the trend regression, we see that the coefficient of variation declines only by 0.00026 per year. Another aspect is that we only regarded absolute beta convergence, ignoring other spatial effects or the impact of regional policy. The latter is also not considered in neoclassical regional growth theory.

Remembering German reunification, we want to test if there are average growth differences between West Germany and East Germany (former German Democratic Republic), which leads to conditional beta convergence. The dataset `G.regions.emp` contains the column `regional`, where the counties are attributed either to West or East Germany, expressed as character string (`"West"` or `"East"`). We need to include our condition into the convergence equation. Thus, we use the `REAT` function `to.dummy()` to create dummy variables (1/0) out of (nominal scaled) variables, and add the indicator for West Germany (1, otherwise 0) to our data:

```
regionaldummies <- to.dummy(G.counties.gdp$regional)
# Creating dummy variables for West/East
# regionaldummies[,1] = East (1/0), regionaldummies[,2] = West (1/0)
G.counties.gdp$West <- regionaldummies[,2]
# Adding the dummy variable for West
```



(a) Absolute beta convergence (b) Sigma convergence

Figure 2: Regional convergence in Germany 2000-2014 (n = 402 counties)

Now, we test for conditional beta and sigma convergence, including the condition "West", again using the `rca()` function, but without plots and using the standard deviation (default setting) instead of the cv for sigma convergence. This time, we save the results in an object:

```
converg_results <- rca (G.counties.gdp$gdppc2000, 2000,
G.counties.gdp[55:68], 2014, conditions = G.counties.gdp[c(70)],
sigma.type = "trend")
# condition variable "West" in column 70
# Store results in "converg_results"
```

The output is:

```
Regional Beta and Sigma Convergence

Absolute Beta Convergence
Model coefficients (Estimation method: OLS)
             Estimate Std. Error t value                  Pr (>|t|)
Alpha       0.0954564  0.0099087   9.634 0.000000000000000000006845
Beta       -0.0071323  0.0009885  -7.215 0.000000000002719258225550
Lambda      0.0005113         NA      NA                          NA
Halflife 1355.7282963         NA      NA                          NA
Model summary
          Estimate F value df 1 df 2           Pr (>F)
R-Squared   0.1152   52.06    1  400 0.000000000002719

Conditional Beta Convergence
Model coefficients (Estimation method: OLS)
             Estimate Std. Error t value          Pr (>|t|)
Alpha       0.0754412  0.0102354   7.371 0.0000000000009872
Beta       -0.0047020  0.0010517  -4.471 0.0000101720129094
West       -0.0053559  0.0009745  -5.496 0.0000000693910790
Lambda      0.0003366         NA      NA                 NA
Halflife 2058.9555949         NA      NA                 NA
Model summary
          Estimate F value df 1 df 2                Pr (>F)
R-Squared   0.1774   43.04    2  399 0.00000000000000001192

Sigma convergence (Trend regression)
           Estimate Std. Error t value      Pr(>|t|)
Intercept  3.895236  0.3267817   11.92 0.00000002264
Time      -0.001764  0.0001628  -10.84 0.00000007041
Model summary
          Estimate F value df 1 df 2       Pr (>F)
R-Squared   0.9003   117.4    1   13 0.00000007041
```

In the `rca()` output, we can compare the results of absolute and conditional beta convergence. In the conditional model, the explained variance increases from $R^2 \approx 0.12$ to $R^2 \approx 0.18$, which indicates an increased explanatory power of the model due to the added condition variable. Both models are statistically significant, also the β values are negative and significant ($p < 0.001$ in both cases). The condition "West" is significant ($t \approx -5.50$, $p < 0.001$) and negative, which means that, on average, the GDP per capita in West German counties grew slower than in East Germany. These results *seem* to support the convergence hypothesis from growth theory, but one should not forget that e.g. political aspects (such as the German and/or EU regional policy) are not considered in this simple analysis.

As we have saved the invisible function output, we can access specific parts of our analysis, such as the regression data for the absolute convergence model:

```
converg_results$betaconv$regdata
# All results in list converg_results
# converg_results contains list betaconv (beta convergence results)
# betaconv contains data frame regdata (regression data)

    ln_initial   ln_growth
1       11.002  0.01997436
2       10.552  0.02980133
3       10.283  0.01794207
4       10.090  0.01763444
5       10.287  0.02361006
...
```

If we want to look at the single sigma values, we can address them via:

```
converg_results$sigmaconv$sigma.trend
# All results in list converg_results
# converg_results contains list sigmaconv (sigma convergence results)
# sigmaconv contains data frame sigma.trend (sigma values)

          years sigma.years
gdp1       2000      0.3646
gdppc2001  2001      0.3662
gdppc2002  2002      0.3618
gdppc2003  2003      0.3606
gdppc2004  2004      0.3592
...
```

4 Specialization of regions and spatial concentration of industries

4.1 Indicators of regional specialization and industry concentration

Specialization of regions or countries and the spatial concentration of industries or firms are phenomena linked to several research fields in regional economics and economic geography: Specialization is a key point in traditional theories of international trade with respect to comparative advantages (Ricardo, 1821) as well as in the generation of the "New Trade Theory" (introduced by Krugman 1979). Spatial clustering of firms or industries due to agglomeration economies is a perennial issue in all spatial economic fields. It especially reemerged in the context of the "New Economic Geography" (e.g. Krugman 1991; Fujita et al. 2001) as well as through the work of Porter (1990) regarding clusters. The common indicators are broadly discussed in Farhauer, Kröll (2014) or Nakamura, Morrison Paul (2009). For studies comparing some different indicators, see e.g. Goschin et al. (2009); Moga, Constantin (2011); Palan (2017).

When looking at the family of indicators of regional specialization and industry concentration, we have to distinguish between indicators for aggregate data, such as regional employment data, and those requiring individual firm data. The first group, compiled in Table 5, can be differentiated into indicators of specialization and indicators of spatial concentration. As both types of agglomeration are closely linked to each other, so are the

Table 5: Coefficients of regional specialization and industry concentration

Indicator	Specialization of region j	Spatial concentration of industry i
Hoover/Balassa	$LQ_{ij} = \frac{e_{ij}/e_i}{e_j/e} \equiv MRCA_{ij} = \frac{e_{ij}/e_j}{e_i/e}$	
	$\overline{LQ_j} = \frac{1}{I}\sum_{i=1}^{I} LQ_{ij}$	$\overline{LQ_i} = \frac{1}{J}\sum_{j=1}^{J} LQ_{ij}$
Extensions:		
O'Donoghue-Gleave	$SLQ_{ij} = \frac{LQ_{ij}-\overline{LQ_i}}{sd(LQ_i)}$	
Tian	$SLLQ_{ij} = \frac{\log(LQ_{ij})-\overline{\log(LQ_i)}}{sd(\log(LQ_i))}$	
Hoen-Oosterhaven	$ARCA_{ij} = \frac{e_{ij}}{e_j} - \frac{e_i}{e}$	
Hoover	$H_j = \frac{1}{2}[\sum_{i=1}^{I} \lvert\frac{e_{ij}}{e_j} - \frac{e_i}{e}\rvert]$ $0 \leq H_j \leq 1$	$H_i = \frac{1}{2}[\sum_{j=1}^{J} \lvert\frac{e_{ij}}{e_i} - \frac{e_j}{e}\rvert]$ $0 \leq H_i \leq 1$
Gini	$G_j = \frac{2}{I^2\bar{R}}\sum_{i=1}^{I} \lambda_i(R_i - \bar{R})$ $0 \leq G_j \leq 1$	$G_i = \frac{2}{J^2\bar{C}}\sum_{j=1}^{J} \lambda_j(C_j - \bar{C})$ $0 \leq G_i \leq 1$
	where: $R_i = \frac{e_{ij}/e_j}{e_i/e}$,	where: $C_j = \frac{e_{ij}/e_i}{e_j/e}$,
	$\bar{R} = \frac{1}{I}\sum_{i=1}^{I} R_i$ and	$\bar{C} = \frac{1}{J}\sum_{j=1}^{J} C_j$ and
	$\lambda_i = 1, ..., I$ $(\lambda_i < \lambda_{i+1})$	$\lambda_j = 1, ..., J$ $(\lambda_j < \lambda_{j+1})$
Krugman $(J = 2,\ I = 2)$	$K_{jl} = \sum_{i=1}^{I} \lvert s^s_{ij} - s^s_{il}\rvert$ $0 \leq K_{jl} \leq 2$	$K_{iu} = \sum_{j=1}^{J} \lvert s^c_{ij} - s^c_{uj}\rvert$ $0 \leq K_{iu} \leq 2$
	where: $s^s_{ij} = \frac{e_{ij}}{e_j}$ and $s^s_{il} = \frac{e_{il}}{e_l}$	where: $s^c_{ij} = \frac{e_{ij}}{e_i}$ and $s^c_{uj} = \frac{e_{uj}}{e_i}$
Extensions:		
Midelfart et al., Vogiatzoglou	$K_j = \sum_{i=1}^{I} \lvert s^s_{ij} - \bar{s}^s_{il}\rvert$ $0 \leq K_j \leq 2$	$K_i = \sum_{j=1}^{J} \lvert s^c_{ij} - \bar{s}^c_{uj}\rvert$ $0 \leq K_i \leq 2$
$(J > 2,\ I > 2)$	where: $s^s_{ij} = \frac{e_{ij}}{e_j}$ and	where: $s^c_{ij} = \frac{e_{ij}}{e_i}$ and
	$\bar{s}^s_{il} = \frac{1}{J-1}\sum_{i}^{J} s^s_{il}$, $l \neq j$	$\bar{s}^c_{uj} = \frac{1}{I-1}\sum_{u}^{I} s^c_{uj}$, $u \neq i$
Duranton-Puga	$RDI_j = \frac{1}{\sum_{i=1}^{I} \lvert\frac{s^s_{ij}}{s_i} - s_i\rvert}$ where: $s^s_{ij} = \frac{e_{ij}}{e_j}$ and $s_i = \frac{e_i}{e}$	
Litzenberger-Sternberg	$CI_{ij} = \frac{IS_{ij}ID_{ij}}{PS_{ij}}$ where $IS_{ij} = \frac{e_{ij}/a_j}{e_i/a}$, $ID_{ij} = \frac{e_{ij}/p_j}{e_i/p}$ and $PS_{ij} = \frac{e_{ij}/b_{ij}}{e_i/b_i}$	

Notes: e_{ij} and e_{il} equal the employment of industry i in regions j and l, respectively, e_i is the total employment in industry i, e_{uj} ist the employment of industry u in region j, e_j is the total employment in region j, e is the total employment in the whole economy, I is the number of industries, J is the number of regions, a_j is the area of region j, a is the total area in the whole economy, p_j is the population in region j, p is the total population, b_{ij} is the number of firms of industry i in region j and b_i is the number of firms in industry i.
Compiled from: Farhauer, Kröll (2014); Hoen, Oosterhaven (2006); Hoffmann et al. (2017); Nakamura, Morrison Paul (2009); O'Donoghue, Gleave (2004); Tian (2013); Schätzl (2000); Störmann (2009)

corresponding indicators. The empirical basis of all those measures is the employment e in industry i in region j, e_{ij}. This employment stock is compared to some reference, mostly including the total employment in region j, e_j, and/or the total employment in industry i, e_i, as well as the all-over employment e. The individual firm level indicators in Table 6 can be segmented into indicators for agglomeration of *one* industry due to localization economies and indicators for the coagglomeration of *different* industries due to urbanization economies.

Table 6: Coefficients of agglomeration and coagglomeration using individual firm data

Indicator	Agglomeration	Coagglomeration
Ellison-Glaeser	$\gamma_i = \frac{G_i - (1-\sum_{j=1}^{J} s_j^2) HHI_i}{(1-\sum_{j=1}^{J} s_j^2)(1-HHI_i)}$ where: $G_i = \sum_{j=1}^{J} (s_{ij}^c - s_j)^2$, $s_{ij}^c = \frac{e_{ij}}{e_i}$, $s_j = \frac{e_j}{e}$ and $HHI_i = \sum_{k=1}^{K} (\frac{e_{ik}}{e_i})^2$ z-standardization: $z_i = \frac{G_i - (1-\sum_{j=1}^{J} s_j^2) HHI_i}{\sqrt{var(G_i)}}$ where: $var(G_i) = 2\Big\{ HHI_i^2 \Big[\sum_{j=1}^{J} s_j^2$ $-2\sum_{j=1}^{J} s_j^3 + (\sum_{j=1}^{J} s_j^2)^2 \Big] -$ $\sum_{k=1}^{K} z_{ik}^4 \Big[\sum_{j=1}^{J} s_j^2 - 4\sum_{j=1}^{J} s_j^3$ $+3(\sum_{j=1}^{J} s_j^2)^2 \Big] \Big\}$	$\gamma^c = \frac{G/(1-\sum_{j=1}^{J} s_j^2) - HHI_U - \sum_{i=1}^{U} \gamma_i s_i^2 (1-HHI_i)}{1-\sum_{i=1}^{U} s_i^2}$ where: $G = \sum_{j=1}^{J} (x_j - s_j)^2$, $x_j = \sum_{i=1}^{U} \frac{e_{ij}}{e_i}$, $s_j = \frac{e_j}{e}$, $s_i = \frac{e_i}{e}$ and $HHI_U = \sum_{i=1}^{U} s_i^2 HHI_i$
Howard et al.		$CL_{ab} = \frac{\sum_{k=1}^{K_a} \sum_{l=1}^{K_b} C_{kl}}{K_a K_b}$ $XCL_{ab} = CL_{ab} - CL_{ab}^{RND}$ where: $C_{kl} = 1$ if firms k and l are located in the same region and $C_{kl} = 0$ otherwise

Notes: e_{ij} is the employment of industry i in region j, e_i is the total employment in industry i, e_j is the total employment in region j, e is the total employment in the whole economy, e_{ik} is the employment of firm k from industry i, k and l are indices for single firms, I is the number of industries, J is the number of regions, U is a subset of all I industries ($U \leq I$), K is the number of firms and K_a and K_b are the numbers of firms in industry a and b.

Compiled from: Farhauer, Kröll (2014); Howard et al. (2016); Nakamura, Morrison Paul (2009)

The most popular indicator is the Location Quotient (LQ), which is attributed to Hoover (1936) and mathematically equivalent to the Revealed Comparative Advantage (RCA) index, developed by Balassa (1965) in the context of international trade. The LQ is utilized in many studies (e.g. Bai et al. 2008; Kim 1995) as well as in the *OECD Territorial Reviews* (OECD, 2019). Following O'Donoghue, Gleave (2004) and Tian (2013), the original formulation can be extended: As the location quotient is not normalized, there is no cut-off value for defining a cluster, which leads to a standardization of the computed values via z-transformation. Hoen, Oosterhaven (2006) developed an additive alternative to the RCA index. The original LQ provides the main mathematical basis for several indicators developed later, such as the spatial Gini coefficients described below.

Some indicators which are known from the context of regional inequality (see Section 2) are also used for the analysis of agglomeration: A modification of the Gini coefficient is used for the spatial concentration of industries as well as regional specialization (e.g. Ceapraz 2008; Wieland, Fuchs 2018). As we can see in the calculation of R_i and C_j, respectively, the spatial Gini coefficient is based on the LQ. Another popular option for analyzing agglomeration is the Hoover coefficient, comparing the structure of an industry/a region to a reference structure of all industries/regions (e.g. Dixon, Freebairn 2009; Jiang et al. 2007). Both indicator types range between zero (no specialization/concentration) and one (total specialization/concentration). Also the Herfindahl-Hirschman index and its derivates are used to measure concentration, specialization and diversification (e.g. Duranton, Puga 2000; Goschin et al. 2009; Lehocký, Rusnák 2016).

Another type of specialization/concentration indicator was introduced by Krugman (1991), originally designed for comparing the specialization of *two* regions. An extension of this indicator was established by Midelfart-Knarvik et al. (2000) for the comparison of regional specialization/industry concentration with respect to the sum or mean of *all*

regions/industries (furthermore used e.g. by Haas, Südekum 2005; Vogiatzoglou 2006). Unlike the Gini- or Hoover-type measures, the Krugman coefficients range between zero (no specialization/concentration) and two (total specialization/concentration).

The cluster index developed by Litzenberger, Sternberg (2006) goes beyond employment data and includes additional information about the industry-specific firm size, population density and region size. It is composed of three parts: the relative industrial stock with respect to industry i and region j, IS_{ij}, the relative industrial density, ID_{ij}, and the relative firm size, PS_{ij}. All three components are modified location quotients. This is done to control for small and monostructural regions, which are identified as clusters otherwise (which is a problem in the original LQ). The cluster index CI_{ij} has a potential range from zero to infinity. This extended indicator is used e.g. by Hoffmann et al. (2017) for the German food processing industry.

The cluster indicators by Ellison, Glaeser (1997) compare the empirical distribution of firms to an arbitrary location pattern where agglomeration economies are absent (often referred to as a *dartboard approach*). Ellison, Glaeser (1997) differentiate between the clustering of firms from one industry (agglomeration) due to localization economies and the clustering of multiple industries (coagglomeration) due to urbanization economies. Their indices also take into account the industry-specific structure of the firms by including the Herfindahl-Hirschman index, HHI_i, for the employment concentration in industry i. This is the reason why individual firm-level data is required for the computation. The Herfindahl-Hirschman indicator is included to control the raw measures of spatial concentration, G_i and G, for firm employment concentration, which occurs especially when there are just a few firms with many employees. The Ellison-Glaeser (EG) index for agglomeration, γ_i, is designed for identifying the clustering of industry i, while the coagglomeration index, γ_c aims at the clustering of a set of U industries, where $U \leq I$. Values of γ equal to zero imply the absence of agglomeration economies, while values above zero indicate positive effects due to spatial clustering. When γ is negative, firm locations are less spatially concentrated than expected on condition of the dartboard approach, which indicates negative agglomeration economies. The EG index is used in several current regional economic studies (e.g. Dauth et al. 2015, 2018; Yamamura, Goto 2018).

In contrast, Howard et al. (2016) argue that agglomeration economies should not be analyzed regarding employment but the firms itself. Their colocation index, CL_{ab}, sums the colocation of K_i and K_q firms from two industries, i and q, controlling for all possible combinations. This colocation measure is compared to a counterfactual location structure constructed via bootstrapping; specifically the arithmetic mean of a number of (e.g. 50) random assignments of the regarded firms to the locations. The value of the resulting excess colocation index, XCL_{ab}, ranges between -1 and 1.

4.2 Application in REAT

4.2.1 REAT functions for regional specialization and industry concentration

Table 7 shows the REAT functions for agglomeration measures based on aggregate (employment) data. All functions require at least information about the employment in one or more regions j in one or more industries i, e_{ij}. The Herfindahl-Hirschman index (function `herf()`) for measuring regional diversity is not displayed as it is used exactly in the same way as described in Section 2, replacing x_i with e_{ij}.

Location quotients for one region and one or more industries are computed by the function `locq()`, including the option for an additive indicator instead of the multiplicative. When calculating the LQ for a set of J regions and I industries, one can use function `locq2()`, which is a kind of batch processing extension of `locq()`. As the dimension of the Litzenberger-Sternberg cluster index is the same as in the LQ (a single value for each combination of region j and industry i), the related functions `litzenberger()` and `litzenberger2()` work in the same way. When using `locq2()` or `litzenberger2()`, the user may choose the type of function output: either a `matrix` with I columns and J rows or a `data frame` with $I * J$ rows.

The Hoover-, Gini- and Krugman-type indicators require the same kind of input data. The `hoover()` function was already explained in Section 2, as it can be also

Table 7: REAT functions for regional specialization and industry concentration

Indicator	REAT function	Mandatory arguments	Optional arguments	Output
Hoover LQ/ Balassa RCA incl. extensions	locq()	vectors or single values of e_{ij} and e_i, single values of e_j and e	LQ method, plot	Single value or matrix with LQ_{ij}
	locq2()	vectors of e_{ij}, industry ID i and region ID j	normalization, output type, remove NAs	matrix or data frame with $I * J$ values of LQ_{ij}
Hoover specialization/ concentration	hoover() (see Section 2)	vectors of e_{ij} and reference vector e_i or e_j	remove NAs	value: H_j or H_i
Gini specialization	gini.spec()	vectors e_{ij} and e_i	plot LC	value: G_j, optional: LC plot
concentration	gini.conc()	vectors e_{ij} and e_j	plot LC	value: G_i, optional: LC plot
Krugman specialization	krugman.spec() (regions j and l)	vectors e_{ij} and e_{il}		value: K_{jl}
	krugman.conc2() (all J regions)	vector e_{ij} and matrix or data frame e_{il}		value: K_j
concentration	krugman.conc() (industries i and u)	vectors e_{ij} and e_{uj}		value: K_{iu}
	krugman.conc2() (all I industries)	vector e_{ij} and matrix or data frame e_{uj}		value: K_i
All at once: specialization	spec()	vectors of e_{ij}, industry ID i and region ID j	remove NAs	matrix with H_j, G_j and K_j (columns) for J regions (rows)
concentration	conc()	vectors of e_{ij}, industry ID i and region ID j	remove NAs	matrix with H_i, G_i and K_i (columns) for I industries (rows)
Duranton-Puga	durpug()	vectors e_{ij} and e_i		value: RDI_j
Litzenberger-Sternberg	litzenberger()	single values of e_ij, e_i, a_j, a, p_j, p, b_{ij} and b_i		value: CI_{ij}
	litzenberger2()	vectors of e_{ij}, industry ID i, region ID j, a_j, p_j and b_{ij}	output type, remove NAs	matrix or data frame with $I * J$ values of CI_{ij}

Source: own compilation.

used for measuring spatial concentration of industries or the specialization of regions with all-over employment vectors, e_i and e_j, respectively, as reference distributions. The spatial Gini coefficients are available through functions `gini.spec()` for regional specialization and `gini.conc()` for spatial concentration. The Krugman coefficients are divided into functions for the comparison of two regions/industries (`krugman.spec()` and `krugman.conc()`, respectively) and for applying all regions/industries as reference (`krugman.spec2()` and `krugman.conc2()`, respectively). The functions `spec()` and `conc()` are wrapper functions providing a convenient way to compute Hoover, Gini and Krugman coefficients of a given set of J regions and I industries at once, e.g. originating from official statistics on regional employment.

Table 8 shows the functions operating on the level of individual firm data. The Ellison-Glaeser (*EG*) indices are available through the functions `ellison.a()` (agglomeration index for industry i) and `ellison.a2()` (agglomeration indices for I industries) as well as `ellison.c()` (coagglomeration index for U industries) and `ellison.c2()` (coagglomeration indices for $I * I - I$ industry combinations). All functions require the firm size (e.g. no. of employees) for the k-th firm from industry i (`numeric vector`) and the

Table 8: REAT functions for agglomeration and coagglomeration using firm data

Indicator	REAT function	Mandatory arguments	Optional arguments	Output
Ellison-Glaeser agglomeration	ellison.a()	vectors of e_{ik}, e_j and region ID j		visible: value γ_i, invisible: matrix with γ_i, G_i, z_i, K_i and HHI_i
	ellison.a2()	vectors e_{ik}, industry ID i and region ID j		visible: values γ_i, invisible: matrix with γ_i, G_i, z_i, K_i and HHI_i, for I industries (rows)
coagglomeration	ellison.c()	vectors e_{ik}, industry ID i and region ID j	vectors e_j and U industries	value: γ^c
	ellison.c2()	vectors e_{ik}, industry ID i and region ID j	vector e_j	matrix with γ^c for $I * I - I$ industry combinations (rows)
Howard et al. colocation	howard.cl()	firm ID k, industry ID i, and region ID j, industries a and b		value: CL_{ab}
excess colocation	howard.xcl()	firm ID k, industry ID i and region ID j, industries a and b, no. of samples		value: XCL_{ab}
	howard.xcl2()	firm ID k, industry ID i and region ID j		matrix with XCL_{ab} for $I * I - I$ industry combinations (rows)

Source: own compilation.

region j the firm is located in. The functions incorporating more than one industry (all except for `ellison.a()`) require a `vector` containing the industry i. The data could e.g. be stored in a `data frame` with at least three columns (firm size, region, industry). Like some of the convergence functions (see Section 3), the EG agglomeration index functions in `REAT` also distinguish between a visible and an invisible output: `ellison.a()` and `ellison.a2()` show the value(s) auf γ_i but return an invisible `matrix` including the raw measure of concentration (G_i), the z-standardized results (z_i) and the related Herfindahl-Hirschman index for industry-specific firm concentration (HHI_i) as well as the number of firms in industry i (K_i).

The Howard-Newman-Tarp coagglomeration measure is distributed over the functions `howard.cl()` (calculation of the colocation index for one pair of industries a and b), `howard.xcl()` (calculation of the excess colocation index for industries a and b) and `howard.xcl2()` (calculation of the excess colocation index for $I * I - I$ combinations of I industries). As this cluster index works with firms instead of employment, we only need a `vector` containing the IDs of the firms k, the corresponding industry i and the region j where the firm is located. When calculating this measure for one pair of industries, the user must state the IDs of industries a and b. Note that calculation time for this index increases heavily with the number of firms and/or industries.

4.2.2 Application example 1: Regional specialization of Göttingen

We use the German classification of economic activities (WZ2008) on the level of 21 sections (A-U) for the classification of industries in the following examples (see Table 9).

Starting with a simple example, we analyze the regional specialization of Göttingen, a city with a population of about 134,000 in Niedersachsen, Germany. The example dataset `Goettingen`, which is included in `REAT`, contains the dependent employees in Göttingen and Germany for 2008 to 2017 in industries A to R (rows 2 to 16; row 1 contains the all-over employment). First, we load the data:

Table 9: Classification of economic activities in Germany, edition 2008 (WZ 2008)

WZ2008 Code	Title
A	Agriculture, forestry and fishing
B	Mining and quarrying
C	Manufacturing
D	Electricity, gas, steam and air conditioning supply
E	Water supply; sewerage, waste management and remediation activities
F	Construction
G	Wholesale and retail trade; repair of motor vehicles and motorcycles
H	Transportation and storage
I	Accommodation and food service activities
J	Information and communication
K	Financial and insurance activities
L	Real estate activities
M	Professional, scientific and technical activities
N	Administrative and support service activities
O	Public administration and defence; compulsory social security
P	Education
Q	Human health and social work activities
R	Arts, entertainment and recreation
S	Other service activities
T	Activities of households as employers; undifferentiated goods-and services-producing activities of households for own use
U	Activities of extraterritorial organisations and bodies

Source: own compilation based on Statistisches Bundesamt (2008).

```
data(Goettingen)
```

Using the `REAT` function `locq()`, we calculate a location quotient for Göttingen with respect to the manufacturing industry ("Verarbeitendes Gewerbe"), which is represented by letter C:

```
locq (Goettingen$Goettingen2017[4], Goettingen$Goettingen2017[1],
Goettingen$BRD2017[4], Goettingen$BRD2017[1])
# Industry: manufacturing (letter C) in row 4
# row 1 = all-over employment

[1] 0.5369
```

The output is simply the LQ value (LQ_{ij}, where i is manufacturing and j is Göttingen). We see that the LQ is very low, indicating that manufacturing is underrepresented in Göttingen as compared to Germany. Now, we calculate LQ values for all industries (A-R), including a simple plot (function argument `plot.results = TRUE`):

```
locq (Goettingen$Goettingen2017[2:16], Goettingen$Goettingen2017[1],
Goettingen$BRD2017[2:16], Goettingen$BRD2017[1],
industry.names = Goettingen$WZ2008_Code[2:16], plot.results = TRUE,
plot.title = "Location quotients for Göttingen 2017")
# all industries (rows 2-16 in the dataset)
```

The output is a `matrix` with one row for each industry:

```
Location quotients
I = 15 industries

            LQ
A   0.08407652
BDE 0.40085663
C   0.53687366
F   0.34366928
G   0.74603541
H   0.67117311
```

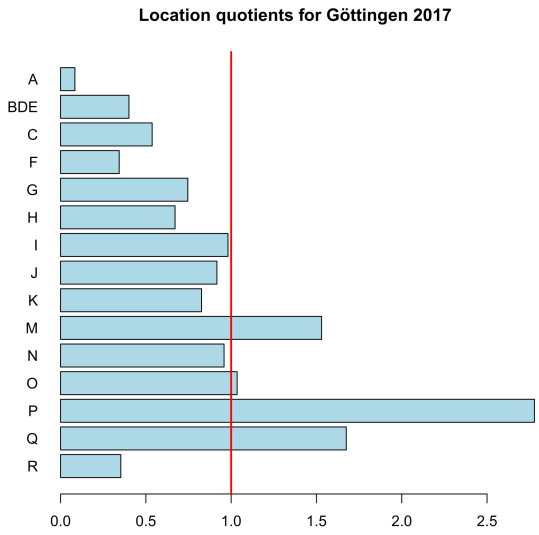


Figure 3: Location quotients for 15 industries in Göttingen

```
I   0.98141916
J   0.91654277
K   0.82650178
M   1.53027645
N   0.95843423
O   1.03509027
P   2.77790858
Q   1.67459967
R   0.35317012
```

The result is plotted in Figure 3. The function plots a vertical line at $LQ_{ij} = 1$ automatically. This is the (only) reference value for the LQ. It indicates a stock of the related industry equal to the whole economy. The highest LQ values can be found for the industries with letters P (education) and Q (health). This is because Göttingen is mainly characterized by a large university (about 30,000 students) with a university hospital with about 7,000 employees.

Now, we want to measure the specialization of Göttingen with a single indicator. First, we simply use the Herfindahl-Hirschman coefficient for both Göttingen and Germany using the function `herf()`:

```
herf(Goettingen$Goettingen2017[2:16])
[1] 0.127314

herf(Goettingen$BRD2017[2:16])
[1] 0.1104567
```

The *HHI* for Göttingen is slightly larger than for Germany, which indicates a higher specialization (or lower economic diversity) of the region. To combine this information in one indicator, we calculate the Hoover coefficient of specialization using the function `hoover()`, where the reference distribution is the German industry structure:

```
hoover(Goettingen$Goettingen2017[2:16], ref = Goettingen$BRD2017[2:16])
[1] 0.2254234
```

We finish our analysis of Göttingen's regional specialization by calculating both the Gini and the Krugman coefficient of regional specialization with the same data, using the `REAT` functions `gini.spec()` and `krugman.spec()`, respectively. Note that, here,

we use the Krugman coefficient to compare the industry structure of Göttingen to the structure of whole Germany (instead of another region within the country, for which this coefficient was originally formulated):

```
gini.spec(Goettingen$Goettingen2017[2:16], Goettingen$BRD2017[2:16])
[1] 0.359852

krugman.spec(Goettingen$Goettingen2017[2:16], Goettingen$BRD2017[2:16])
[1] 0.4508469
```

There seems to be some specialization in Göttingen, but, unfortunately, we do not have any real reference value to interpret the results.

4.2.3 Application example 2: Identifying clusters in Germany using aggregate data

In this example, we will compute indicators of regional specialization and industry concentration for a set of J regions and I industries at once. We load the included test dataset `G.regions.industries` containing employment and firms on the level of $I = 17$ industries (WZ2008 codes B-S) and $J = 16$ regions ("Bundesländer") in Germany:

```
data(G.regions.industries)
```

The number of employees in the column `emp_all` includes dependent employees and self-employed persons. The classification code of industries (see Table 9) can be found in column `ind_code`, while the region code (abbreviation of the region's official name) is in column `region_code`. First, we want to detect the spatial concentration of the 17 industries in Germany by calculating Hoover, Gini and Krugman coefficients for all industries at once, applying the `REAT` function `conc()` which is a wrapper function for the mentioned indicators. We save our output in the `matrix` object `conc_i`:

```
conc_i <- conc (e_ij = G.regions.industries$emp_all,
industry.id = G.regions.industries$ind_code,
region.id = G.regions.industries$region_code)
```

The output is:

```
Spatial concentration of industries
I = 17 industries, J = 16 regions

              H i        G i        K i
WZ08-B 0.22959050 0.42334831 0.45675385
WZ08-C 0.09933363 0.17047620 0.26813759
WZ08-D 0.07754576 0.12509360 0.16260016
WZ08-E 0.11972072 0.16742909 0.20369011
WZ08-F 0.07676634 0.15357575 0.16996098
WZ08-G 0.03034962 0.05471323 0.07977056
WZ08-H 0.06006957 0.11921850 0.10076748
WZ08-I 0.05177262 0.09939075 0.11450791
WZ08-J 0.10230712 0.22605802 0.24450967
WZ08-K 0.08982871 0.17610712 0.20565974
WZ08-L 0.09798632 0.16784764 0.17472656
WZ08-M 0.06490185 0.14760918 0.14931991
WZ08-N 0.06714816 0.08575299 0.09053327
WZ08-P 0.03019678 0.05053848 0.07043586
WZ08-Q 0.04679962 0.06170335 0.06406058
WZ08-R 0.09424708 0.16748405 0.17023603
WZ08-S 0.04507988 0.07246697 0.06441360
```

The function returns a `matrix` with 17 rows (one for each industry) and three columns: `H i` is the Hoover coefficient, `G i` is the Gini coefficient and `K i` is the Krugman coefficient for industry i. We cannot interpret or compare all of these results, but we may pick out some findings: The strongest spatial concentration is found with respect to mining

and quarrying (WZ08-B), no matter which indicator is regarded, which may be interpreted with "natural advantages" due to the spatial distribution of mineral resources in Germany. Services (such as retailing) as well as education and health are least concentrated, as these industries are bound to regional demand and/or their locations are regulated by policy and planning authorities.

At a first glance, the three indicators seem to produce similar results. Now, we want to test the similarity between Hoover, Gini and Krugman coefficients of concentration. As we saved our result `matrix`, we now calculate Pearson correlation coefficients (r) for each pair of indicators using the basic `R` function `cor()`, which is implemented in the `stats` package (included automatically in any `R` release). The function is applied to the three columns of `conc_i`, producing a $3 * 3$ correlation matrix:

```
cor(conc_i[,1:3])
          H i       G i       K i
H i 1.0000000 0.9676518 0.9527747
G i 0.9676518 1.0000000 0.9681770
K i 0.9527747 0.9681770 1.0000000
```

As we can see, each combination of the three indicators shows a strong positive correlation (H_i vs. G_i: $r \approx 0.97$, H_i vs. K_i: $r \approx 0.95$, G_i vs. K_i: $r \approx 0.97$). At least in this context, we may conclude that these indicators are interchangeable. However, we have to recognize that the analysis presented here is on a large-scale regional level (German "Bundesländer") and all of the mentioned indicators are affected by the *modifiable areal unit problem*, which means that the results depend on the aggregation unit in the analysis (see e.g. Dapena et al. 2016 for a discussion of this effect).

Now, we do exactly the same with respect to regional specialization of the 16 regions, using the same data. Analogously, we use the wrapper function `spec()` for calculating Hoover, Gini and Krugman coefficients of regional specialization, also saving the resulting `matrix`:

```
spec_j <- spec (e_ij = G.regions.industries$emp_all,
industry.id = G.regions.industries$ind_code,
region.id = G.regions.industries$region_code)
```

The output is:

```
Specialization of regions
I = 17 industries, J = 16 regions

          H j        G j        K j
BB 0.11530353 0.20632682 0.18555259
BE 0.17891265 0.29040841 0.34552331
BW 0.08024011 0.10300695 0.22675612
BY 0.05008135 0.07659148 0.16019603
HB 0.09502615 0.18563500 0.17467291
HE 0.05494422 0.12160142 0.11282696
HH 0.16413456 0.22616814 0.33190321
MV 0.13270849 0.18974606 0.22056868
NI 0.03772799 0.08237225 0.07972852
NW 0.02940091 0.05997505 0.07181569
RP 0.04793147 0.07432361 0.12036513
SH 0.08901907 0.11384295 0.15994524
SL 0.05726933 0.11921727 0.15071159
SN 0.05400855 0.10643512 0.10341280
ST 0.08821395 0.21120287 0.15280711
TH 0.08234046 0.13902924 0.17720208
```

The strongest specialization can be found in the city states Berlin (BE) and Hamburg (HH), while Niedersachsen (NI) and Nordrhein-Westfalen (NW) show the lowest values in all three indicators. As already mentioned in the concentration example, we have to remember the large-scale aggregation unit. If we used smaller scale units (e.g. counties like in Section 3.2.2), our results would surely be more differentiated. Again, we check the correlation between the indicators:

```
cor(spec_j[,1:3])
          H j       G j       K j
H j 1.0000000 0.9179127 0.9322604
G j 0.9179127 1.0000000 0.7907841
K j 0.9322604 0.7907841 1.0000000
```

Again, we find a strong positive correlation between the Hoover coefficient and both Gini and Krugman coefficient (H_j vs. G_j: $r \approx 0.92$, H_j vs. K_j: $r \approx 0.93$), while the third Pearson correlation coefficient is a little lower, but still showing the same direction (G_j vs. K_j: $r \approx 0.79$).

Now we check for clusters in a combination of a specific industry and a specific region. First, we calculate location quotients for the dataset `G.regions.industries` using the `REAT` function `locq2()`. Here, the optional function argument `LQ.norm` could be used for computing z-standardized location quotients according to O'Donoghue, Gleave (2004) (`LQ.norm = "OG"`) or z-standardized values of the natural-logged LQs according to Tian (2013) (`LQ.norm = "T"`). However, we produce the original LQs, since we need exactly the same columns as in the examples above:

```
locq2(e_ij = G.regions.industries$emp_all,
G.regions.industries$ind_code, G.regions.industries$region_code)
```

The output is a `matrix` with J rows and I columns:

```
Location quotients
I = 17 industries, J = 16 regions

              BB          BE        BW        BY        HB        HE
WZ08-B 2.5314363 0.04030901 0.6607950 0.8078054 0.0000000 0.3735773
WZ08-C 0.6857231 0.37224900 1.3968652 1.1902785 0.7863570 0.8580352
WZ08-D 1.1736475 0.46721079 1.0861988 0.8343784 0.9179718 0.9627955
WZ08-E 1.7945685 1.30128835 0.5896526 0.7137388 1.2393228 0.8532203
WZ08-F 1.5997778 0.77160121 0.9070096 1.0280409 0.6212923 0.8927681
WZ08-G 0.9550127 0.83133221 0.9492523 0.9879826 0.9013193 1.0006321
WZ08-H 1.3212794 0.87982228 0.8189666 0.8664163 1.7692815 1.2208728
WZ08-I 1.0379426 1.35561299 0.9132949 1.0390886 0.9904308 0.9571339
WZ08-J 0.5625876 1.78334039 1.0316114 1.1550764 1.0577107 1.1407078
WZ08-K 0.6529329 0.76630600 0.9329930 1.1058890 0.7825178 1.6710583
WZ08-L 1.1088846 2.13220960 0.7310014 0.8633894 1.3254723 1.1132939
WZ08-M 0.7366238 1.39880205 1.0337139 1.0265993 1.1202532 1.1457770
WZ08-N 1.2571301 1.24261162 0.7977054 0.8486971 1.2938161 1.0525912
WZ08-P 0.9052976 1.38842157 0.9649289 0.9252245 1.0563169 1.0085207
WZ08-Q 1.1540423 1.09329902 0.8695241 0.9079679 0.9544891 0.8980680
WZ08-R 1.0656945 2.55595102 0.8518192 0.8220540 1.3196613 0.8651451
WZ08-S 1.1409373 1.32596177 0.8626829 0.9092125 1.1396616 1.0528184
              HH        MV        NI        NW        RP        SH
WZ08-B 0.6029388 0.6235796 1.4987086 1.4595767 1.0371236 0.5078145
WZ08-C 0.4781934 0.6230156 0.9512438 0.9312325 1.0822678 0.7082513
WZ08-D 0.4332870 1.0118838 0.9932719 1.2139740 0.9349679 1.1248685
WZ08-E 1.1442005 1.5642257 1.0645497 1.0408356 0.9886860 1.0707585
WZ08-F 0.5432163 1.2716537 1.0969756 0.8735506 1.1134885 1.1043449
WZ08-G 1.0654315 0.9485377 1.0758977 1.0612190 1.0111274 1.2456100
WZ08-H 1.4958610 1.1243732 1.0409143 0.9961224 0.9633972 1.0112557
WZ08-I 1.0634066 1.7574637 1.0227196 0.8750205 1.1121264 1.2483966
WZ08-J 1.9266913 0.4751473 0.6716376 0.9830609 0.8122058 0.7496925
WZ08-K 1.5175078 0.5383900 0.9108456 1.0205798 0.8879292 0.8355178
WZ08-L 1.5871838 1.3034074 0.8040270 0.9928161 0.7774500 1.1980553
WZ08-M 1.6293913 0.6897571 0.8693026 1.0366589 0.7764558 0.7905498
WZ08-N 1.2530608 1.2484353 0.9675147 1.0659893 0.8026181 0.9727871
WZ08-P 0.9422739 0.9966228 1.0888054 0.9846351 1.0976178 0.9540262
WZ08-Q 0.8564604 1.2893168 1.0728412 1.0595648 1.0418460 1.1662290
WZ08-R 1.4914564 1.0500685 0.9204586 0.9611539 0.8498053 1.0418794
WZ08-S 0.8055128 1.1158184 0.9965451 1.0283571 1.1658852 1.1455178
```

```
              SL        SN        ST        TH
WZ08-B 0.2826284 1.2746172 2.4654331 0.7140637
WZ08-C 1.1752810 0.9867417 0.9297172 1.1849897
WZ08-D 1.1465539 1.0637093 1.2642787 0.8607578
WZ08-E 0.9555581 1.4457486 1.8251853 1.6042935
WZ08-F 0.9016858 1.3794286 1.4104724 1.3481005
WZ08-G 1.0370901 0.8787739 0.9172598 0.8661184
WZ08-H 0.8851047 1.0476688 1.2012430 0.8944907
WZ08-I 0.9111877 0.9496370 0.9020582 0.8644257
WZ08-J 0.7133587 0.7717704 0.4874344 0.6869177
WZ08-K 1.0082983 0.6620719 0.6133933 0.6316347
WZ08-L 0.7018816 1.1395422 1.0111694 0.8511896
WZ08-M 0.8060753 0.8459317 0.6627301 0.6814339
WZ08-N 1.0751749 1.1656467 1.2796548 1.1093251
WZ08-P 0.9147874 0.9658590 0.9798932 0.9710576
WZ08-Q 1.0760969 1.0475595 1.1401680 1.0628602
WZ08-R 0.7631263 1.1419135 0.8329295 0.8582919
WZ08-S 0.8840741 0.9774726 0.9257397 1.0923137
```

These $I * J = 17 * 16 = 272$ coefficients are too much information. Thus, we calculate them again using the optional argument `LQ.output = "df"`, which produces a `data frame` with $I * J$ rows and three columns (`j_region`: ID of region j, `i_industry`: ID of industry i and `LQ`: location quotient LQ_{ij}). We save the results in the object `lqs`:

```
lqs <- locq2(e_ij = G.regions.industries$emp_all,
G.regions.industries$ind_code, G.regions.industries$region_code,
LQ.output = "df")
```

As we forego an inspection of these singe values, the results are not displayed here. Instead, we only deal with the five highest LQs in our results (the "top five"). We sort the resulting `data frame` decreasing and take a look at the first five rows:

```
lqs_sort <- lqs[order(lqs$LQ, decreasing = TRUE),]
# Sort decreasing by size of LQ

lqs_sort[1:5,]

    j_region i_industry       LQ
33        BE     WZ08-R 2.555951
1         BB     WZ08-B 2.531436
239       ST     WZ08-B 2.465433
28        BE     WZ08-L 2.132210
111       HH     WZ08-J 1.926691
```

The highest LQ is found for the arts, entertainment, and recreation sector (WZ08-R) in the German capital Berlin. Note that this result is congruent with several studies about the "creative class", showing a large stock of "creative" employment in Berlin (e.g. Martin 2015). We also find a strong concentration of mining and quarrying in two Eastern regions, Brandenburg and Sachsen-Anhalt. Note that the LQ is a *relative* measure with respect to the total regional employment as well as the total industry-specific employment and the employment in the whole economy, not considering other aspects of industry or spatial structure.

These deficiencies should be overcome with the Litzenberger-Sternberg cluster index, also taking into account area, population and firm size. This additional data is also included in our current dataset (columns `area_sqkm`, `pop` and `firms`). The functions `litzenberger()` and `litzenberger2()` work equivalently to `locq()` and `locq2()`. To compute cluster indices for all $I * J$ combinations, we use the function `litzenberger2()`:

```
litzenberger2(G.regions.industries$emp_all,
G.regions.industries$ind_code, G.regions.industries$region_code,
G.regions.industries$area_sqkm, G.regions.industries$pop,
G.regions.industries$firms)
```

Like in `locq2()`, the default output is a `matrix` with I rows and J columns:

```
Litzenberger-Sternberg cluster indices
I = 17 industries, J = 16 regions

              BB           BE        BW        BY        HB        HE
WZ08-B 0.5736692   0.05611505 0.8041813 1.1073446       NaN 0.4745084
WZ08-C 0.1610679   3.24717820 2.6250805 1.2043415  5.119669 1.0603087
WZ08-D 0.2213627   1.37720778 1.7043505 1.4178208  4.172162 0.8541359
WZ08-E 0.8810260  10.14891585 0.8235517 0.6890213  6.705744 1.1427285
WZ08-F 0.7142888  11.36434353 1.2372108 0.9442221  3.498921 1.1225087
WZ08-G 0.2707787  12.10626625 1.3903532 0.9404677  7.136205 1.3361060
WZ08-H 0.4386878  13.26265081 1.0955747 0.7982074 22.656924 1.8358272
WZ08-I 0.2672336  26.60020727 1.4098657 0.9633029  8.481338 1.2880210
WZ08-J 0.1130326  59.24931837 1.4342037 1.2393579  8.683998 1.9024826
WZ08-K 0.1524825  10.28664194 1.4774980 1.1692461  6.650213 2.4739044
WZ08-L 0.2564814  56.65943460 0.9594093 0.8695636 11.839900 1.6099929
WZ08-M 0.1685895  39.30149403 1.5306799 1.0110472  9.410498 1.7302434
WZ08-N 0.4232166  26.91532975 1.0228326 0.7471872 10.796027 1.5265846
WZ08-P 0.2043023  25.30839556 1.3656409 0.9509028  7.322709 1.4627871
WZ08-Q 0.3445630  21.86956483 1.1770297 0.7873877  7.850886 1.2163624
WZ08-R 0.2450932 104.73565741 1.0839767 0.7821779 10.555369 1.0489672
WZ08-S 0.2891132  24.71310833 1.3435576 0.9594882  9.893688 1.5421903
              HH         MV        NI        NW        RP        SH
WZ08-B  2.319611 0.16000177 1.5004530 2.074735 1.1951266 0.3119091
WZ08-C  4.000104 0.11993714 0.5036838 2.010757 0.9646351 0.4008598
WZ08-D  1.679371 0.20954802 0.8848956 2.001708 0.6016715 1.2989105
WZ08-E 11.129156 0.44055556 0.6476797 1.915196 0.8616891 0.8101087
WZ08-F  5.526083 0.45291220 0.6276791 1.656384 0.8973162 0.8055662
WZ08-G 15.627090 0.22665565 0.6898359 2.377365 0.8115817 0.9008287
WZ08-H 44.371836 0.32192237 0.6420146 1.980491 0.7087341 0.7205961
WZ08-I 14.795885 0.59842705 0.6335126 1.731963 1.0572996 1.0361389
WZ08-J 59.720584 0.05895184 0.2905769 2.139796 0.5142953 0.4427180
WZ08-K 26.189623 0.12112900 0.5648050 1.953963 0.7104929 0.5758818
WZ08-L 33.175443 0.25167830 0.5228248 2.223854 0.5349641 0.8587588
WZ08-M 44.433527 0.11657637 0.4440959 2.297308 0.4961611 0.4456119
WZ08-N 23.255195 0.31231467 0.5271293 2.413351 0.5537510 0.7348068
WZ08-P 14.294075 0.24008982 0.7203953 2.022971 0.9704975 0.6937792
WZ08-Q 13.211918 0.38626491 0.6877487 2.260518 0.7770980 0.8426253
WZ08-R 44.256214 0.20066782 0.4508862 2.190752 0.5287290 0.6694273
WZ08-S 14.195156 0.27631480 0.5364848 1.941470 0.7996291 0.9238816
              SL        SN         ST        TH
WZ08-B 0.3673090 1.1179110 1.49064630 0.4562730
WZ08-C 1.8348713 1.0311722 0.32128528 0.7892360
WZ08-D 0.9643263 0.4534029 0.29391783 0.2086042
WZ08-E 1.9939461 1.5554125 1.09973945 1.1815726
WZ08-F 1.3245152 1.8167263 0.68075807 1.0313749
WZ08-G 1.7528078 0.7644695 0.31005922 0.4313306
WZ08-H 1.1099151 0.9297053 0.42973342 0.4881643
WZ08-I 1.7871163 0.7119567 0.29998391 0.4012910
WZ08-J 0.8984679 0.4927030 0.08046178 0.2087174
WZ08-K 1.5505928 0.5980588 0.21942805 0.3160432
WZ08-L 0.8170723 0.8226773 0.21044244 0.2886068
WZ08-M 1.0137151 0.6489868 0.15797219 0.2493458
WZ08-N 1.3940298 1.2492658 0.42285503 0.5935329
WZ08-P 1.2331390 0.7800531 0.31406013 0.4389675
WZ08-Q 1.8551266 1.0749000 0.48353914 0.5854787
WZ08-R 0.8477702 0.8666478 0.21562997 0.2919300
WZ08-S 1.6138955 0.8600519 0.34624522 0.5369766
```

Note that there is a value equal to `NaN`, which means "not a number", due to a division by zero; this is because there is no mining and quarrying (WZ08-B) in Bremen (HB). However, we take a look at the "top five" again:

```
lss <- litzenberger2(G.regions.industries$emp_all,
G.regions.industries$ind_code, G.regions.industries$region_code,
G.regions.industries$area_sqkm, G.regions.industries$pop,
G.regions.industries$firms, CI.output = "df")

lss_sort <- lss[order(lss$CI, decreasing = TRUE),]

lss_sort[1:5,]

    j_region i_industry        CI
33        BE     WZ08-R 104.73566
111       HH     WZ08-J  59.72058
26        BE     WZ08-J  59.24932
28        BE     WZ08-L  56.65943
114       HH     WZ08-M  44.43353
```

Again, we find the largest cluster value for the arts and entertainment sector in Berlin. Also the other four highest indicators are discovered in the largest city states Berlin and Hamburg, especially with respect to the information and communication industry (WZ08-J) and other knowledge-intensive services. Obviously, the results of the Litzenberger-Sternberg index differ in a noticeable way from those of the LQ, which can be attributed to the consideration of other spatial aspects, especially controlling for the size of the regions.

4.2.4 Application example 3: Identifying clusters using micro-data

In our last example about agglomerations, we use the Ellison-Glaeser indices and the Howard-Newman-Tarp colocation index, which both require individual firm data. As this kind of micro-data is sensitive and, of course, not available in official statistics, we have to use fictional data from the textbook by Farhauer, Kröll (2014).

At first, we compute the Ellison-Glaeser agglomeration index for one industry i, γ_i. We use the `REAT` function `ellison.a()`, which is designed for this purpose and requires three `vectors`: the size (employment) of firm k, e_{ik}, the IDs of the regions j each firm is located in, and the total regional employment, e_j. The numerical example in Farhauer, Kröll (2014), Table 14.11, contains ten firms in three regions (Wien, Linz, and Graz). We simply compile the data from the original table into separate `vectors`:

```
region <- c("Wien", "Wien", "Wien", "Wien", "Wien", "Linz",
"Linz", "Linz", "Linz", "Graz")
# regions (Austrian cities)
emp_firm <- c(200,650,12000,100,50,16000,13000,1500,1500,25000)
# employment of the ten firms
emp_region <- c(500000,400000,100000)
# employment of the three regions
```

Now, we apply `ellison.a()` to this data:

```
ellison.a (emp_firm, emp_region, region)
[1] 0.05990628
```

The EG agglomeration index of $\gamma_i \approx 0.06$, which is, by the way, the same result as in the textbook, indicates a stronger clustering than expected from a dartboard approach. Since this data is fictional, we refrain from interpreting this result.

The `REAT` package contains the dataset `FK2014_EGC`, which is compiled from the numerical example in Farhauer, Kröll (2014), Tables 14.14 to 14.17. There are $k = 42$ firms from $I = 4$ industries (clothing trade, forestry, textiles dyeing and textiles trade) in $J = 3$ regions (1, 2 and 3). We load this example data:

```
data(FK2014_EGC)
```

We compute γ_i for all industries in the dataset. This can be done with the function `ellison.a2()`, which requires `vectors` containing the size of firm k, the corresponding industry i, and region j. We save the results in the object `ega`:

```
ega <- ellison.a2 (FK2014_EGC$emp_firm, FK2014_EGC$industry,
FK2014_EGC$region)
```

Here, we see the output of the function:

```
Ellison-Glaeser Agglomeration Index
K = 42 firms, I = 4 industries, J = 3 regions

                    Gamma i
Clothing trade  -0.09379384
Forestry         0.16838003
Textiles dyeing -0.08012539
Textiles trade  -0.13040134
```

We see a strong clustering of the forestry industry, which is attributed to localization economies, but spatial avoidance in the three other industries. The visible output of `ellison.a2()` contains the γ_i values only, but the invisible `matrix` output also includes the other information referring to the *EG* agglomeration index:

```
ega
                    Gamma i         G i       z i K i      HHI i
Clothing trade  -0.09379384 0.017909653 -0.5025978  11 0.13124350
Forestry         0.16838003 0.088262934  1.3660878  13 0.09240553
Textiles dyeing -0.08012539 0.027764811 -0.3801644   9 0.14559983
Textiles trade  -0.13040134 0.002734966 -0.7663541   9 0.12208059
```

When looking at the forestry industry, we also see a high standardized value ($z_i \approx 1.37$) and a relatively low firm concentration ($HHI_i \approx 0.09$).

In the next step, we compute the *EG* coagglomeration index, γ^c, for the same data using the function `ellison.c()`. This function requires the same information as `ellison.a2()` plus the total employment in the regarded regions (column `emp_region`):

```
ellison.c (FK2014_EGC$emp_firm, FK2014_EGC$industry,
FK2014_EGC$region, FK2014_EGC$emp_region)

[1] 12.0729
```

Congruent with the calculation in Farhauer, Kröll (2014), the function returns $\gamma^c \approx 12.07$. This value is very large, which indicates urbanization economies in this fictional example.

If we want to analyze the coagglomeration of industry pairs instead, we may use the function `ellison.c2()`, which requires the same data:

```
ellison.c2 (FK2014_EGC$emp_firm, FK2014_EGC$industry,
FK2014_EGC$region, FK2014_EGC$emp_region)
```

The output is a `matrix` with $I * I - I$ rows (one for each industry pair, omitting the combination of the same industry i):

```
Ellison-Glaeser Co-Agglomeration Index
K = 42 firms, I = 4 industries, J = 3 regions

                                Gamma c
Forestry-Clothing trade        1.382257
Textiles dyeing-Clothing trade 2.465609
Textiles trade-Clothing trade  2.067766
Clothing trade-Forestry        1.382257
Textiles dyeing-Forestry       1.570292
Textiles trade-Forestry        1.336020
Clothing trade-Textiles dyeing 2.465609
Forestry-Textiles dyeing       1.570292
Textiles trade-Textiles dyeing 2.294259
Clothing trade-Textiles trade  2.067766
Forestry-Textiles trade        1.336020
Textiles dyeing-Textiles trade 2.294259
```

If we want to focus on firm numbers instead of employment size, we may compute the Howard-Newman-Tarp excess colocation index, which is included in `REAT` through the functions `howard.cl()` for one colocation index for one pair of industries, `howard.xcl()` for the corresponding excess colocation index and `howard.xcl2()` for all combinations of $I * I$ industries. Subsequent to the numerical example above, we calculate XCL_{ab} for all industry pairs in the dataset `FK2014_EGC`, where the firm ID of k is stored in the column `firm`:

```
howard.xcl2 (FK2014_EGC$firm, FK2014_EGC$industry,
FK2014_EGC$region)
# this takes some seconds
```

The output has the same structure as the output from `ellison.c2()`:

```
Howard-Newman-Tarp Excess Colocation Index
K = 42 firms, I = 4 industries, J = 3 regions

                                    XCL
Forestry-Clothing trade        0.01902098
Textiles dyeing-Clothing trade 0.02909091
Textiles trade-Clothing trade  0.02020202
Clothing trade-Forestry        0.02377622
Textiles dyeing-Forestry       0.03282051
Textiles trade-Forestry        0.03589744
Clothing trade-Textiles dyeing 0.02707071
Forestry-Textiles dyeing       0.02666667
Textiles trade-Textiles dyeing 0.02814815
Clothing trade-Textiles trade  0.02101010
Forestry-Textiles trade        0.01743590
Textiles dyeing-Textiles trade 0.03012346
```

We see that the index by Howard et al. (2016) is structured differently than the indicators presented above: Although they are based on exactly the same data, the value for forestry and clothing trade ($XCL \approx 0.019$) is *not* equal to the value for clothing trade and forestry ($XCL \approx 0.024$). Why? The XCL_{ab} is the difference between the colocation index, CL_{ab}, and the mean of a set of bootstrap samples, CL_{ab}^{RND} (see Table 6). These random samples are drawn again each time a XCL value is computed, consequently, also the XCL value changes.

5 Proximity and accessibility

5.1 Distance-based measures of accessibility and proximity using individual point-level data

In this chapter, we mix two different concepts of indicators, accessibility and spatial proximity (see Table 10), both frequently used especially in the context of GIS (geographic information systems). Both concepts are discussed together because they have two aspects in common: 1) they are based on the geographical distance between point locations, in particular, the distance between an origin point i or several origin points ($i = 1, ..., n$) and one or more destination points j ($j = 1, ..., m$), and 2) for the calculation, they require geocoded (with geographical coordinates) individual point data.

One popular indicator of accessibility is the Hansen accessibility, developed by Hansen (1959) in the context of land use theory. The basic idea is that "accessibility" equals the sum of opportunities outgoing from a specific origin i. These opportunities are spread over a set of m locations ($j = 1, ..., m$). The summation is weighted with the distance between i and the j-th location. This distance, no matter how measured (e.g. street distance, Euclidean distance, driving time) is assumed to be perceived in a nonlinear way, which is operationalized by a nonlinear distance decay function (a.k.a. distance impedance function or response function), e.g. power, exponential or logistic. A similar concept was introduced by Harris (1954) attempting to model the market potential of

Table 10: Accessibility and proximity indicators using point-level data

Indicator	Non-normalized	Normalized	
Accessibility/Market potential			
Harris	$M_j = \sum_{i=1}^{n} O_i d_{ij}^{-1}$ $0 \leq M_j \leq \infty$		
Hansen	$A_i = \sum_{j=1}^{m} O_j f(d_{ij})$ $i \neq j$ $0 \leq A_i \leq \infty$	$A_i^* = \frac{\sum_{j=1}^{m} O_j f(d_{ij})}{\sum_{j=1}^{m} O_j}$ $i \neq j$ $0 \leq A_i^* \leq 1$	
	where: $f(d_{ij}) = d_{ij}^{-\lambda}$ or $f(d_{ij}) = e^{-\lambda * d_{ij}}$ or $f(d_{ij}) = \frac{1}{1+e^{-\lambda_1+\lambda_2 d_{ij}}}$		
Proximity			
Count within buffer	$N_i = \sum_{i=1}^{n} I(d_{ij} \leq t)$ $i \neq j$		
Weighted count within buffer	$N_i^w = \sum_{i=1}^{n} I(d_{ij} \leq t) O_j$ $i \neq j$		
Ripley	$K_t = \frac{1}{\lambda} \sum_{i=1}^{n} \frac{I(d_{ij} \leq t)}{n}$ $i \neq j$	$L_t = \sqrt{\frac{K_t}{\pi}}$ $i \neq j$	$H_t = L_t - t$ $i \neq j$
	$E(K_t) = \pi t^2$	$E(L_t) = t$	$E(H_t) = 0$
	where: $\lambda = \frac{n}{A}$		

Notes: d_{ij} is the distance from origin location i ($i = 1, ..., n$) to destination location j ($j = 1, ..., m$), O_j is a variable quantifying the size of destination j, t is a maximum search radius and $I(d_{ij} \leq t)$ is the indicator function taking the value of $I = 1$ if $d_{ij} \leq t$, and $I = 0$ otherwise.
Compiled from: Kiskowski et al. (2009); Krider, Putler (2013); Peña Carrera (2002); Pooler (1987); Reggiani et al. (2011); Smith (2016)

locations. If we replace the inverse distance weighting in the Harris indicator with another type of distance weighting, we see that both concepts are mathematically equivalent. The only difference is that the Harris indicator is conceptualized from the supplier's perspective j (e.g. market potential of a retail store) and the Hansen accessibility takes the demand location i as a starting point (Pooler, 1987; Reggiani et al., 2011). As these indicators are dimensionless and range from zero to infinity, a normalization with a range from zero to one can be computed by weighting the results with the opportunities without distance correction.

This accessibility/potential concept can be used in the regional economic context e.g. to quantify the over-regional job potential (e.g. Wieland, Fuchs 2018) or the clustering of point locations of a specific type, such as retail stores (e.g. Larsson, Öner 2014). The most common application of these indicators may be the context of transport economics and transport geography (e.g. Albacete et al. 2017).

In the GIS context, spatial proximity can be measured using concentric zones within a radius of t (buffers) around point i, where the number of the j points within this radius is counted (Longley et al., 2005). A systematic analysis of spatial proximity or cluster patterns is possible using Ripley's K function (Ripley, 1976). It compares empirical point counts with expected values from a random spatial point process based on a Poisson distribution. Ripley's K computes empirical values for each distance band with a maximum distance of t, which can be compared to the expected value. A more comprehensible (and linear) interpretation is provided when normalizing the K function in the form of the L or H function. Also, confidence intervals for the expected values can be calculated by bootstrapping (Kiskowski et al., 2009; Smith, 2016). All of these measures are based on a simple indicator function, $I(d_{ij} \leq t)$, which takes the value of $I = 1$ if point j is within a distance of t from point i or not ($I = 0$). Originating from natural sciences, especially Ripley's K is frequently used when analyzing location patterns in spatial economic contexts, such as the clustering of retail stores (e.g. Krider,

Table 11: REAT functions for accessibility and proximity on the point level

Indicator	REAT function	Mandatory arguments	Optional arguments	Output
Distance matrix	dist.mat()	data frame(s) with start points i (ID, lat, lon) and end points j (ID, lat, lon), distance unit	$i \neq j$	data frame with from, to, from-to and distance d_{ij} (distance matrix)
Buffer	dist.buf()	data frame(s) with start points (ID, lat, lon) and end points (ID, lat, lon), max. distance t, distance unit	$i \neq j$, sum O_j at endpoints	list with distance matrix (data frame) and count table (data frame)
Hansen/ Harris	hansen()	distance matrix (data frame with start points i and end points j as well as distance d_{ij} and O_j), weighting functions, parameters λ and γ	distance constant, max. distance t, $i \neq j$	data frame with origins i and accessibility A_i
Ripley	ripley()	data frame with points (ID, lat, lon), total area A, max. distance t, number of distance intervals	local K values, confidence intervals no. of samples, significance level, plot (K, L or H)	visible: matrix with t, K_t, $E(K_t)$, $K_t - E(K_t)$, L_t and H_t for each distance interval, invisible: matrix (as described above) and optional: matrices with local K values and confidence intervals

Source: own compilation.

Putler 2013) or other types of firms assumed to be connected in a network (e.g. Espa et al. 2010).

5.2 Application in REAT

5.2.1 REAT functions for accessibility and proximity on the point level

Table 11 shows the `REAT` functions for the accessibility and proximity methods described above. A simple Euclidean distance matrix for georeferenced points (`data frame` with latitude and longitude) can be calculated using the function `dist.mat()`. The function `dist.buf()` computes a "count points within buffer", where also a weighting, O_j, can be summarized (e.g., if the destination points are cities of a given population, one could count the number of cities within 50 kilometers and their corresponding population). The latter function uses `dist.mat()`, thus, it is not necessary to create a distance matrix before.

The same is the case for the function `ripley()`, which calculates Ripley's K function for georeferenced data (`data frame` with lat/lon) and a given number of distance intervals up to a maximum distance of t. The differences between the empirical values, K_t, and the expected values, $E(K_t)$, as well as the normalizations (L_t and H_t) are calculated and returned automatically. Optionally, local K values for each distance interval and corresponding confidence intervals are computed. These confidence intervals are based on bootstrapping with a given number of samples (default: 100) on a given significance level (the default value is $\alpha = 0.05$, which leads to confidence intervals of a range from $\alpha/2 = 2.5\%$ to $1-\alpha/2 = 97.5\%$). Note that the plot of the K function (or, when desired, L or H function) provides a graphical and more intuitive interpretation of the analyzed point pattern, especially when including confidence intervals.

When calculating the Hansen accessibility (or the Harris market potential) with `hansen()`, a distance matrix including the opportunities, O_j, is required. This can be, of course, done with `dist.mat()` (if straight-line distances are sufficient), but also with any other software creating distance matrices (and any type of transport costs indicator). In `hansen()`, the user may choose between a power, exponential or logistic distance decay function. Optionally, the normalized Hansen accessibility is returned additionally.

5.2.2 Application example 1: Location analysis of medical practices

In the example in Section 2.2.2, we dealt with small-scale regional inequality in health care in South Lower Saxony, Germany. We have seen that e.g. psychotherapists are more spatially clustered than general practitioners (GPs). Returning to this topic, we want to use proximity and accessibility measures for determining the market potential (in the sense of the Harris model) of these health care locations. Obviously, there are different location patterns of general practitioners and psychotherapists. In the related study, there was evidence that psychotherapists are not just clustered but clustered within some districts of larger cities (Wieland, Dittrich, 2016). In the German health care planning system, the market potential of medical practices is the main determinant of the official authorization to be included into the allocation system of health insurance, while psychotherapists are assumed to need quite larger market areas than GPs (Kassenärztliche Bundesvereinigung, 2013). Consequently, our research hypothesis is that the population potential of psychotherapists is larger than that of general practitioners.

We use the same test data as in the mentioned example, containing the health locations (`GoettingenHealth1`) and the corresponding settlements (`GoettingenHealth2`). We load both `R` datasets:

```
data(GoettingenHealth1)
data(GoettingenHealth2)
```

Table `GoettingenHealth1` contains 617 locations, whose ID is stored in the column `location`. Columns `lat` and `lon` contain the latitude and longitude, respectively, while the corresponding location type can be found in column `type` (`phys_gen`: general practitioners, `psych`: psychotherapists, `pharm`: pharmacies). As the following applications may be time-consuming, we extract the general practitioners from `GoettingenHealth1` and draw a random sample of ten doctor's practices:

```
physgen <- GoettingenHealth1[GoettingenHealth1$type == "phys_gen",]
# general practitioners: column "type" is equal to "phys_gen"
physgen_sample <- physgen[sample(nrow(physgen),10),]
# random sampling of ten general practitioners
```

Now, we want to summarize the population potential of these health locations in a 1,000 meters buffer. We apply the function `dist.buf()` to the sample data `physgen_sample` and sum up the local population of the districts within this distance (column `pop` in `GoettingenHealth2`):

```
physgen_pot <- dist.buf (physgen_sample, "location", "lat", "lon",
GoettingenHealth2, "district", "lat", "lon", bufdist = 1000,
ep_sum = "pop")
# counting all districts within a radius of 1000 meters
# and summing the corresponding population
```

We calculate the arithmetic mean of all ten potentials:

```
mean2(physgen_pot$count_table$sum_pop)
[1] 8027.7
```

On average, the ten GP practices have a population potential of about 8,028 inhabitants. One problem related to the buffer technique is the lack of distance weighting: All origin points up to a given distance are included completely, while all points above 1,000 meters are ignored. Thus, we repeat estimating the population potential using the Hansen accessibility. At first, we need an origin-destination matrix (distance matrix) from the origin points to the sampled GP locations. We use the function `dist.mat()` and merge the returned distance matrix with the population values from `GoettingenHealth2`:

```
physgen_od <- dist.mat(GoettingenHealth2, "district", "lat", "lon",
physgen_sample, "location", "lat", "lon")
# creating OD matrix from all districts to the
# sampled general practitioners

physgen_od <- merge (physgen_od, GoettingenHealth2,
by.x = "from", by.y = "district")
# merging with GoettingenHealth2 to include the
# population values of the districts
```

Then, we use the function `hansen()` to calculate the Hansen accessibility (used in the sense of the Harris market potential model) for each GP location in `physgen_od`.

The required columns in this dataset are the IDs of the GP locations (`to`), the IDs of the districts (`from`) and the population of the districts (`pop`) as well as the distances calculated above (`distance`). Finally, we have to set a distance weighting (which has an important influence in all types of spatial interaction models like this). For this purpose, we fall back on the results of a study by Fülöp et al. (2011): Based on empirical patient's choice of doctor, they estimated distance decay functions in spatial interaction models (Huff model) for several types of physicians. For GPs, an exponential distance decay function with $\lambda = -0.28$ was found to fit the empirical data best. To set a distance decay function type and the related weighting(s), the function arguments `dtype` and `lambda` must be used. We save the results under the name `physgen_hansen`:

```
physgen_hansen <- hansen (physgen_od, "to", "from", "pop",
"distance", dtype = "exp", lambda = -0.28)
# calculating Hansen accessibility for the ten
# sampled general practitioners
```

The output of the `hansen()` function is:

```
Hansen Accessibility

J = 420 locations with mean attractivity = 1138.486
I = 10 origins with mean transport costs = 28.07581
Attractivity weighting (pow) with Gamma = 1
Distance weighting (exp) with Lambda = -0.28

     to accessibility
1  1103     24267.054
2  1171     17629.564
3  1206      9581.732
4  1220      9213.407
5   197     10023.854
6   301      6489.571
7   600     69676.232
8   755     66921.123
9   966     13154.921
10  974      3666.171
```

Again, we calculate the arithmetic mean of the distance-weighted market potentials:

```
mean2(physgen_hansen$accessibility)
[1] 23062.36
```

The average population potential of the ten GPs is equal to 23,063 inhabitants.

As we want to compare the market potential of GPs and psychotherapists, we repeat the same analysis for them, now in the "fast mode", leaving out most comments, as the functions and commands are exactly the same as above, only applied to psychotherapists.

```
psychgen <- GoettingenHealth1[GoettingenHealth1$type == "psych",]

psych_sample <- psychgen[sample(nrow(psychgen),10),]

psych_pot <- dist.buf (psych_sample, "location", "lat", "lon",
GoettingenHealth2, "district", "lat", "lon", bufdist = 1000,
ep_sum = "pop")

mean2(psych_pot$count_table$sum_pop)
[1] 12245.88
```

The calculation of Hansen accessibility is different from the one for GPs with respect to the assumed distance reaction of the (potential) clients: For psychotherapists, Fülöp et al. (2011) found a distance impedance which is considerably smaller than for GPs (and any other type of doctor), resulting in a weighting parameter of $\lambda = -0.11$ in the exponential decay function:

```
psych_od <- dist.mat(GoettingenHealth2, "district", "lat", "lon",
psych_sample, "location", "lat", "lon")

psych_od <- merge (psych_od, GoettingenHealth2,
by.x = "from", by.y = "district")

psych_hansen <- hansen (psych_od, "to", "from", "pop",
"distance", dtype = "exp", lambda = -0.11)


Hansen Accessibility

J = 420 locations with mean attractivity = 1138.486
I = 10 origins with mean transport costs = 25.56756
Attractivity weighting (pow) with Gamma = 1
Distance weighting (exp) with Lambda = -0.11

     to accessibility
1  1031      43415.63
2  1213      39226.26
3   179      33491.41
4   313      51228.41
5   506     147887.43
6   786     147969.39
7   791     147971.80
8   811     148021.51
9   872     147475.57
10  922      42424.51

mean2(psych_hansen$accessibility)
[1] 94911.19
```

We see that the average population potential of the sampled psychotherapists on the 1,000 meters buffer level is equal to 12,246 inhabitants, which is about one third more than for GPs. The Hansen/Harris market potential of psychotherapists of about 94,911 persons is a fourfold increase compared to the GPs. We have to remember that the last result is not only a matter of location but also due to a lower assumed distance decay. However, the population potential of the sampled psychotherapists is obviously higher than the potential of the GPs, which can be attributed to a different location pattern, where psychotherapists are more clustered within larger city districts.

5.2.3 Application example 2: Clustering of health service providers

We stick to the example of health care locations. As we have found different degrees of regional inequality with respect to suppliers (Section 2.2.2) and of market potentials

(Section 5.2.2), we now analyze the clustering patterns of health service providers. In South Lower Saxony there is nearly the same number of psychotherapists (118) and pharmacies (120), but we should not expect their location patterns to be similar or even equal. Following the results above, we hypothesize that psychotherapists are more spatially clustered than pharmacies (as we already know about clustering with respect to districts in the former case and we can expect an avoidance tendency in the latter case due to a high degree of substitutability).

For this analysis, we compute Ripley's K with the `REAT` function `ripley()`. Before going on, we have to prepare two things: First, we load the required dataset. Then, we must calculate the total area of the study area manually (here: in square meters).

```
data (GoettingenHealth1)

area_goe <- 1753000000
# area of Landkreis Goettingen (sqm)
area_nom <- 1267000000
# area of Landkreis Northeim (sqm)
area_gn <- area_goe+area_nom
```

Now, we compute Ripley's K for the pharmacies only, which means processing only those locations in `GoettingenHealth1` which are pharmacies (`type == "pharm"`). We set our maximum search radius equal to $t = 30000$ (function argument `t.max`), divided into 300 distance intervals (`t.sep`), resulting in distance steps of 100 meters. As we want to check for a significant deviation from a random spatial pattern, we instruct the function to construct confidence intervals (`ci.boot = TRUE`) using the default settings ($\alpha = 0.05$, 100 bootstrapping samples). We also plot the results (default function argument: `K.plot = TRUE`) to inspect our results graphically. Here, we plot K_t, which is also the default setting (if the user wants to plot L_t or H_t instead, the function argument `Kplot.func` has to be changed to "`L`" or "`H`", respectively):

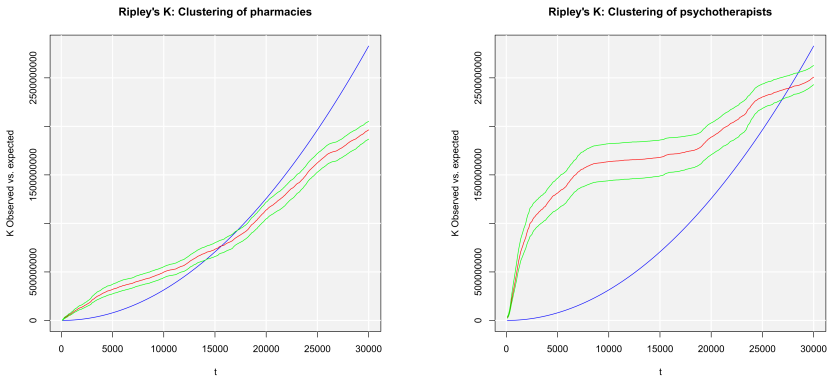
```
ripley(GoettingenHealth1[GoettingenHealth1$type == "pharm",],
"location", "lat", "lon", area = area_gn, t.max = 30000, t.sep = 300,
K.local = TRUE, ci.boot = TRUE, ci.alpha = 0.05, ciboot.samples = 100,
plot.title = "Ripley's K: Clustering of pharmacies")
```

The output is a `matrix` with six columns and one row for each distance interval. Thus, we skip the full output here:

```
Ripley's K
n = 120 points

     t <=       K t exp        K t  Kt-Kt exp       L t         H t
1     100      31415.93    3355556    3324140  1033.492   933.49238
2     200     125663.71   12583333   12457670  2001.349  1801.34940
3     300     282743.34   25586111   25303368  2853.824  2553.82412
4     400     502654.82   32297222   31794567  3206.326  2806.32580
5     500     785398.16   39008333   38222935  3523.739  3023.73923
...
```

We repeat the computation of Ripley's K for the psychotherapists:

```
ripley(GoettingenHealth1[GoettingenHealth1$type == "psych",],
"location", "lat", "lon", area = area_gn, t.max = 30000, t.sep = 300,
K.local = TRUE, ci.boot = TRUE, ci.alpha = 0.05, ciboot.samples = 100,
plot.title = "Ripley's K: Clustering of psychotherapists")
```



(a) Pharmacies (n=120) (b) Psychotherapists (n=118)

Figure 4: Plots of the Ripley-K function with confidence intervals

The output is (also truncated):

```
Ripley's K
n = 118 points

     t <=       K t exp         K t    Kt-Kt exp       L t          H t
1     100      31415.93    30798621   30767205.2  3131.055  3031.055025
2     200     125663.71    48583740   48458076.6  3932.516  3732.516350
3     300     282743.34    80249928   79967184.8  5054.141  4754.141421
4     400     502654.82   132737719  132235064.2  6500.133  6100.132940
5     500     785398.16   202143062  201357664.2  8021.480  7521.479612
...
```

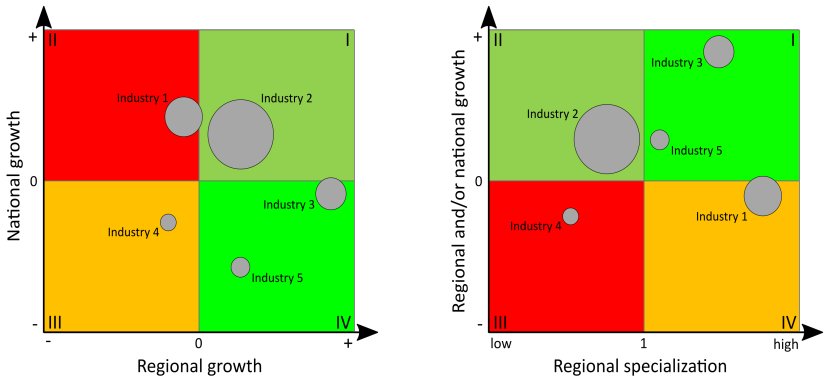
The graphical output is shown in Figures 4a (pharmacies) and 4b (psychotherapists), respectively. The expected value of K_t is plotted as blue line, while the empirical K_t values are red and the corresponding confidence intervals are colored in green (These colors are the default values in `ripley()` and can be changed by the function arguments `lcol.exp` and `lcol.emp`, respectively). As we have nearly the same number of points in both cases within the same field area, a direct comparison seems reasonable. Obviously, both types of locations show a significant spatial clustering: Also the pharmacies are more clustered than expected on condition of complete spatial randomness up to a distance of about 15,000 meters. We have to remember that also the population is already clustered (see Section 2.2.2) and the spatial distribution of pharmacies may follow this pattern. However, the clustering of psychotherapists exceeds this level enormously, especially within smaller distances up to about 8,000 meters. In conclusion, the psychotherapists are more spatially clustered than pharmacies.

6 Analysis and prognosis of regional growth

6.1 Tools and models concerning regional growth

6.1.1 Analyzing regional growth: shift-share analysis and portfolio matrix

Aspects of regional growth have already been discussed in the context of regional convergence in Section 3. The identification of clusters was the topic of Section 4. Combining some aspects of both, this section presents a collection of tools and models concerning regional growth with respect to industries. Like the indicators in Section 4, these techniques are of high significance especially in the context of local economic policy and municipal business promotion activities, aiming at e.g. strengthening a city's or region's competitiveness, defining its profile or increasing the number of jobs (Dinc, 2015; Nischwitz et al., 2017). Inspired by Farhauer, Kröll (2014) and congruent with the mathematical formulations in Section 4, we calculate on the basis of local/regional employment, e_{ij},



(a) Regional growth portfolio

(b) Regional growth & specialization portfolio

Figure 5: Regional economic portfolio matrix

which is the number of employees of industry i in region j. Its growth from time t to time $t + y$ can be operationalized as an absolute value ($\Delta e_{ij} = e_{ij_{t+y}} - e_{ij_t}$) or as a relative growth ($\Delta e_{ij}^{rg} = e_{ij_{t+y}}/e_{ij_t}$) or as a (percentage) growth rate ($\Delta e_{ij}^{gr} = e_{ij_{t+y}}/e_{ij_t} - 1$).

The first technique described is the regional economic portfolio matrix, originating from the portfolio matrix in marketing, developed by the Boston Consulting Group (BCG) for the identification of growing and declining business fields of firms (Henderson, 1973). However, this technique can also be applied to several regional economic contexts (Baker et al., 2002; Howard, 2007). Here, we present a portfolio matrix which compares the growth in *one* region with the growth in a superordinate reference region (e.g. whole economy). When using the matrix in this way, it is a plot of the growth rate with respect to industry i in the region (Δe_{ij}^{gr}) on the x axis and the corresponding growth in the reference region (Δe_{i}^{gr}) on the y axis (see Figure 5a). The size of the points for each industry may be the total size of employment in the region (e_{ij}) to reflect the absolute relevance of the i-th industry. The plot is segmented into four quadrants, differentiated with respect to positive or negative growth rates. As implied by the colors of the quadrants, they can be interpreted as follows: Quadrant I (top right) contains the industries growing in both the region and the whole economy (or any other reference region). Quadrant II (top left) shows all industries growing in the whole economy but shrinking in the regarded region, which may indicate significant locational handicaps. Quadrant III (bottom left) includes all industries shrinking in the region as well as in the whole economy. Quadrant IV (bottom right) shows the special case of "star" industries, indicating that these industries grow in the regarded region while shrinking in the whole economy. Note that this segmentation (and the corresponding interpretation) differs from the original BCG matrix.

Another variant of the portfolio matrix, which was developed in the context of designing the `REAT` package, is shown in Figure 5b. Combining the aspects of regional specialization (see Section 4) and regional growth, we can plot the location quotient as an indicator of local specialization on the x axis, while plotting an industry-specific growth indicator on the y axis. For identifying "growing" industries, there are at least three options of operationalization: We can plot the industry-specific regional growth rate (Δe_{ij}^{gr}) on the y axis (which is on the x axis in the portfolio matrix in Figure 5a) or the industry-specific national rate (Δe_{i}^{gr}) or, if we want to show regional growth in relation to national growth, the quotient of industry-specific regional and national growth rates ($\Delta e_{ij}^{gr}/\Delta e_{i}^{gr}$). In quadrant I, we see now all industries overrepresented in the region (in terms of the location quotient) as well as growing on the regional/national level. Quadrant II shows all industries underrepresented in the region but growing as well. In quadrants III and IV, we can identify all industries with negative growth rates, which are underrepresented or overrepresented, respectively.

Table 12: Shift-share analysis: Dunn and Gerfin type

Component	Dunn-type (absolute)	Gerfin-type (index)
	$\Delta e_j = e_{j_{t+y}} - e_{j_t} = n_{j_{t,t+y}} + m_{j_{t,t+y}} + c_{j_{t,t+y}}$	
Net total shift	$t_{t+y} = e_{j_{t+y}} - e_{j_t} - n_{j_{t,t+y}} = m_{j_{t,t+y}} + c_{j_{t,t+y}}$	$t_{t+y} = m_{j_{t,t+y}} c_{j_{t,t+y}} = \frac{\frac{e_{j_{t+y}}}{e_{j_t}}}{\frac{e_{t+y}}{e_t}}$
static (two time periods t and $t+y$)		
National share	$n_{j_{t,t+y}} = e_{j_t} \frac{e_{t+y}}{e_t} - e_{j_t}$	$n_{j_{t,t+y}} = 1$ (omitted)
Industrial mix	$m_{j_{t,t+y}} = \sum_{i=1}^{I} e_{ij_t} \frac{e_{i_{t+y}}}{e_i} - e_{j_t} \frac{e_{t+y}}{e_t}$	$m_{j_{t,t+y}} = \frac{\sum_{i=1}^{I} e_{ij_t} \frac{e_{i_{t+y}}}{e_i}}{e_{j_t} \frac{e_{t+y}}{e_t}}$
Regional share	$c_{j_{t,t+y}} = \sum_{i=1}^{I} e_{ij_t} (\frac{e_{ij_{t+y}}}{e_{ij_t}} - \frac{e_{i_{t+y}}}{e_{i_t}})$	$c_{j_{t,t+y}} = \frac{\frac{e_{ij_{t+y}}}{e_{ij_t}}}{\sum_{i=1}^{I} e_{ij_t} \frac{e_{i_{t+y}}}{e_i}}$
dynamic (T time periods, while $T > 2$)		
National share	$n_{j_{t,T}} = \sum_{t=1}^{T} e_{j_t} \frac{e_{t+1}}{e_t} - e_{j_t}$	
Industrial mix	$m_{j_{t,T}} = \sum_{t=1}^{T} \sum_{i=1}^{I} e_{ij_t} \frac{e_{i_{t+1}}}{e_i} - e_{j_t} \frac{e_{t+1}}{e_t}$	
Regional share	$c_{j_{t,T}} = \sum_{t=1}^{T} \sum_{i=1}^{I} e_{ij_t} (\frac{e_{ij_{t+1}}}{e_{ij_t}} - \frac{e_{i_{t+1}}}{e_{i_t}})$	
industry-specific		
National share	$n^i_{j_{t,t+y}} = e_{ij_t} \frac{e_{i_{t+y}}}{e_{i_t}} - e_{ij_t}$	
Regional share	$c^i_{j_{t,T}} = e_{ij_t} (\frac{e_{ij_{t+y}}}{e_{ij_t}} - \frac{e_{i_{t+y}}}{e_{i_t}})$	
prognosis for time period z		
Employment		$\Delta e_{ij_{t+z}} = e_{ij_{t+y}} (\frac{e^P_{i_{t+z}}}{e_{i_{t+y}}}) c_{j,t+y}$

Notes: e_{j_t} is the employment in region j at time t, e_{ij_t} is the employment of industry i in region j at time t, e_t is the total employment in the whole economy at time t, e_{i_t} is the total employment in industry i, y and z are numbers of time periods added to t ($z > y$), T is the number of regarded time periods and I is the number of industries.
Compiled from: Farhauer, Kröll (2014); Haynes, Parajuli (2014); Schätzl (2000); Schönebeck (1996); Spiekermann, Wegener (2008); Barff, Knight (1988)

A well-established model of regional growth is the shift-share analysis, which is, although developed independently from the portfolio matrix, closely linked to the concept presented above. The original shift-share analysis was introduced by Dunn Jr. (1960) and given a theoretical foundation by Casler (1989). Parallelly and independently, Gerfin (1964) developed a variant of shift-share analysis, which is more popular in the German-speaking regional economic science. Both concepts have been extended in several ways. Table 12 shows the basics of shift-share analysis with respect to "Dunn" and "Gerfin" type. As there are several ways of formulating the shift-share formulae and calling the particular elements of the shift-share analysis, the description here is based on the mathematical formulations in Farhauer, Kröll (2014) and the terms used in Haynes, Parajuli (2014).

The basic idea of shift-share analysis is the decomposition of regional growth into components, recognizing that single economic regions are embedded into and influenced by a larger regional system, normally the whole economy, just called "the nation" hereinafter: The (employment or e.g. gross value added) growth of industry i in region j from time t to time $t+y$ can be attributed to 1) a national trend, which means the economic climate in the whole system of regions, 2) the all-over growth or decline of the regarded industries and 3) the industry-specific performance of the region, which is linked to locational advantages or disadvantages. The first component is called *national share* and reflects the growth in region j that *would* have occurred if region j *would* have developed exactly as the nation. The second component is the *industrial mix*, represent-

ing the aggregated industry-specific growth in region j if the regarded industries *would* have developed like in the whole economy, adjusted by the national effect. The third component is the *regional share*, which is the "residuum" of the first two components; this share of growth is attributed to locational advantages (or disadvantages), showing the regional growth adjusted by national and industry effects (Farhauer, Kröll, 2014; Haynes, Parajuli, 2014).

The Dunn-type models deal with absolute growth (Δe_{ij} or Δe_j), which is the sum of all shift-share components, and a *net total shift*, which is the sum of the industrial mix and the regional share (as these components are region-specific). Thus, this technique is also called the "difference method". The Gerfin-type approaches express growth in terms of indices, while the net total shift for region j is the result of a multiplication of the industrial mix index and the regional share index, resulting in the alternative denomination "index method" (Schätzl, 2000).

Several extensions have been developed for the Dunn-type shift-share analysis (Haynes, Parajuli, 2014). One regular application calculates a shift-share analysis for each industry i in region j (instead of computing components for the whole region), while skipping the industrial mix effect. A main contribution was the dynamic shift-share analysis by Barff, Knight (1988). It extended the Dunn model by dealing with growth within a longitudinal cut of T years. Other extensions of the Dunn-type technique provide a deeper differentiation of the three components, which are regarded as correlated (e.g. Arcelus 1984; Esteban-Marquillas 1972).

6.1.2 Commercial area prognosis

Also developed independently in the context of German urban planning, a commercial area prognosis deals with an absolute (assumed) employment growth (Δe_{ij}) over T years, which is used to forecast the required commercial area within a city or region j up to time T. Note that "commercial area" represents the type of urban area which is used by specific economic activities, especially industrial plants, and/or designated for this purpose in municipal land-use plans. This technique is a demand-side approach, as it derives the required commercial area from the (expected) demand for it (Bonny, Kahnert, 2005). See Table 13 for the calculation of two types of commercial area prognosis based on employment growth.

The basic model called *GIFPRO* (German abbreviation for "Gewerbe- und Industrieflächenbedarfsprognose", roughly translated: prognosis of future demand of commercial area) was developed by Stark et al. (1981). The usual procedure is to estimate – starting from the current employment – the future industry-specific employment in region j. This number of employees is weighted by the industry-specific shares of workers usually located in commercial areas and multiplied by a resettlement rate (sq_{ij} percent of employees from industry i are resettled in one time period) and a relocation rate (rq_{ij} percent of employees from industry i are relocated in one time period) as well as a reutilization rate (ru_{ij} percent of employees from industry i will be located at reused commercial area). This "commercial area-relevant" employment is weighted with an areal index, a_{ij} (commercial area per employee), to compute the commercial area for industry i in region j for one time period t. The expected commercial area is summed over all I industries (A_{j_t}) and, finally, over all T years and I industries (A_{j_T}) (Bonny, Kahnert, 2005; Planungsgruppe MWM, 2009).

A significant extension was developed in the context of establishing a land-use plan for Dresden: The *TBS-GIFPRO* (German abbreviation for "Trendbasierte und standortspezifische Gewerbe- und Industrieflächenbedarfsprognose", roughly translated: trend-based and location-specific prognosis of future demand of commercial area) technique (Deutsches Institut für Urbanistik GmbH, Spath + Nagel (GbR), 2010). It includes a stochastic approach for forecasting employment as well as other region-specific data. The employment prognosis is done using a trend regression model (employment against time) based on past empirical employment data for region j (mostly from official employment statistics) which are used for forecasting future employment. For each i industry, a single regression model is estimated, where the function type is not pre-defined but chosen e.g. based on the explained variance (R^2) and/or plausibility considerations.

Table 13: Commercial area prognosis

Prognosis	GIFPRO	TBS-GIFPRO
Employment	$e^A_{ij_t} = \left[\left(e_{ij_{t0}} \frac{a_i}{100} \frac{sq_{ij}}{100}\right) + \left(e_{ij_{t0}} \frac{a_i}{100} \frac{rq_{ij}}{100}\right)\right] - \left(e_{ij_{t0}} \frac{ru_{ij}}{100}\right)$	$e^A_{ij_t} = \left[\left(e_{ij_t} \frac{a_i}{100} \frac{sq_{ij}}{100}\right) + \left(e_{ij_t} \frac{a_i}{100} \frac{rq_{ij}}{100}\right)\right] - \left(e_{ij_t} \frac{ru_{ij}}{100}\right)$ where: $e_{ij_t} = f(t) = a + bt$ or $f(t) = at^b$ or $f(t) = ae^{bt}$ or $f(t) = \frac{e^{MAX}_{ij}}{1+e^{-a+bt}}$
Areal index	pre-defined: ai_{ij}	empirical estimation: $ai_{ij} = \frac{A_{ij}}{e_{ij}}$
Commercial area	$A_{ij_t} = e^A_{ij_t} ai_{ij}$ $A_{j_t} = \sum_{i=1}^{I} A_{ij_t}$ $A_{j_T} = \sum_{i=1}^{I} \sum_{t=1}^{T} A_{ij_t}$	

Notes: $e^A_{ij_t}$ is the (expected) number of employees of industry i in region j which is located in commercial areas at time t, $e_{ij_{t0}}$ is the employment of industry i in region j at start time $t0$ (empirical value), e_{ij_t} is the (expected) employment of industry i in region j at time t, a_i is the share of employees in industry i which is located in commercial areas, sq_{ij} is the resettlement rate with respect to industry i in region j in one time period, rq_{ij} is the relocation rate with respect to industry i in region j in one time period, ru_{ij} is the reutilization rate with respect to industry i in region j in one time period, ai_{ij} is the areal index with respect to industry i in region j (commercial area per employee), A_{ij_t} is the (expected) commercial area for industry i in region j at time t, A_{j_t} is the (expected) commercial area in region j at time t and A_{j_T} is the sum of the (expected) commercial area in region j over all T time periods.
Compiled from: Bonny, Kahnert (2005); CIMA Projekt + Entwicklung GmbH et al. (2011); Deutsches Institut für Urbanistik GmbH, Spath + Nagel (GbR) (2010); Planungsgruppe MWM (2009); Mulligan (2006); Vallée et al. (2012)

The function may be linear (which seems unrealistic) or not: Deutsches Institut für Urbanistik GmbH, Spath + Nagel (GbR) (2010) use linear and exponential functions. However, from the growth perspective, also a logistic function may be applied (see Mulligan 2006 for a discussion of logistic growth with respect to population). If possible, the areal index and, maybe, other parameters are also estimated empirically for the specific region j (e.g. via firm-level surveys and/or official statistical data).

6.2 Application in REAT

6.2.1 REAT functions for analyzing and forecasting regional growth

Table 14 shows the functions for the analysis of regional growth as implemented in `REAT`. Table 15 presents the functions related to commercial area prognosis. All of these functions require at least current employment data for each industry in the regarded region j, e_{ij}, which may be a single `numeric vector` or the column of a `data frame` or `matrix`. Another similarity of all mentioned functions is the optional argument of the industry names (or codes). If no industry names are stated by the user (default function argument: `industry.names = NULL`), the industries are numbered consecutively. With respect to the function output, all regional growth functions distinguish between a visible and an invisible output (see e.g. Section 3), where the main results are returned automatically and the details are included in the invisible output (mostly a `list` with several entries of type `matrix`).

The portfolio matrix (growth portfolio and growth-specialization portfolio, respectively) can be plotted using the functions `portfolio()` and `locq.growth()`, respectively. The different techniques of shift-share analysis are distributed over five functions (`shift()`, `shiftd()`, `shifti()`, `shiftid()` and `shiftp()`). The usage of portfolio and shift-share functions is similar: In any case, the user needs industry-specific employment data for the regarded region and the reference region (e.g. whole economy) for at least two time periods (e.g. years).

Table 14: REAT functions for analyzing regional growth

Model	REAT function	Mandatory arguments	Optional arguments	Output
Growth portfolio matrix	portfolio()	vectors of e_{ij_t} and $e_{ij_{t+y}}$ and vectors of e_{i_t} and $e_{i_{t+y}}$ or matrix/data frame with e_{ij_t} and e_{i_t} for T years, point size (e.g. $e_{ij_{t+y}}$)	point size factor, industry names	visible: plot, invisible: growth rates (matrix)
Growth and specialization portfolio matrix	locq.growth()	vectors of e_{ij_t} and $e_{ij_{t+y}}$ and vectors of e_{i_t} and $e_{i_{t+y}}$ or matrix/data frame with e_{ij_t} and e_{i_t} for T years, point size (e.g. $e_{ij_{t+y}}$)	point size factor, industry names	visible: plot, invisible: list with portfolio data (matrix), LQ_{ij} (matrix) and growth rates (matrix)
Shift-share analysis	shift()	vectors of e_{ij_t} and $e_{ij_{t+y}}$, vectors of e_{i_t} and $e_{i_{t+y}}$	shift-share method (default: Dunn), industry names, plot components, plot portfolio	visible: matrix with components, invisible: list with components (matrix), growth (matrix) and shift method (char), optional: plot(s)
dynamic	shiftd()	vectors of $e_{ij_{t0}}$ and $e_{i_{t0}}$, matrix/data frame with e_{ij_t} and e_{i_t} for T years	shift-share method (default: Dunn), industry names, plot components, plot portfolio	visible: matrix with annual components, invisible: list with components (matrix), annual components (matrix), growth (matrix) and shift method (char), optional: plot(s)
industry-specific	shifti()	vectors of e_{ij_t} and $e_{ij_{t+y}}$, vectors of e_{i_t} and $e_{i_{t+y}}$	shift-share method (default: Dunn), industry names, plot components, plot portfolio	visible: matrix with industry components, invisible: list with components (matrix), industry components (matrix), growth (matrix) and shift method (char), optional: plot(s)
industry-specific and dynamic	shiftid()	vectors of $e_{ij_{t0}}$ and $e_{i_{t0}}$, matrix/data frame with e_{ij_t} and e_{i_t} for T years	shift-share method (default: Dunn), industry names, plot components, plot portfolio	visible: matrix with industry components, invisible: list with components (matrix), industry components (matrix), growth (matrix) and shift method (char), optional: plot(s)
prognosis	shiftp()	vectors of e_{ij_t} and $e_{ij_{t+y}}$, vectors of e_{i_t} and $e_{i_{t+y}}$, vector of $e^P_{i_{t+z}}$	industry names, plot	visible: matrix with industry components, invisible: list with industry employment prognosis (matrix), components (matrix), industry components (matrix), growth (matrix) and shift method (char), optional: plots

Source: own compilation.

Table 15: REAT functions for commercial area prognosis

Model	REAT function	Mandatory arguments	Optional arguments	Output
GIFPRO	gifpro()	vectors of e_{ij}, a_i, sq_{ij}, rq_{ij} and ai_{ij}, time interval, time base	vector of ru_{ij}, industry names, type of output	visible: total commercial area and (optional) annual values, invisible: list with components (matrices), annual and all-over results (list with two matrices)
TBS-GIFPRO	gifpro.tbs()	vectors of e_{ij_t} for T years, a_i, sq_{ij}, rq_{ij} and ai_{ij}, time interval, time base, trend function types	vector of ru_{ij}, industry names, type of output, employment forecast only	visible: total commercial area and (optional) annual values, invisible: list with components (matrices), annual and all-over results (list with two matrices), industry-specific forecast model results (list with I matrices)

Source: own compilation.

All functions for shift-share analysis (except for shift-share prognosis with `shiftp()`) provide three variants of calculation of the components: The classical Dunn method (default function argument `shift.method="Dunn"`), the Dunn extension by Esteban-Marquillas (1972) (`shift.method="Esteban"`) producing four components instead of three, and the Gerfin method (`shift.method="Gerfin"`). When calculating a dynamic shift-share analysis, the user must choose the function `shiftd()`. Industry-specific components are returned by the function `shifti()`. With `shiftid()` one can combine both approaches. Here, it is important to recognize that the function structure allows a combination of e.g. industry-specific and dynamic components while calculating the components from the Esteban-Marquillas extension of shift-share analysis. Additionally, the shift-share functions may plot a portfolio matrix (function argument `plot.portfolio = TRUE`), allowing portfolio and shift-share analysis at once.

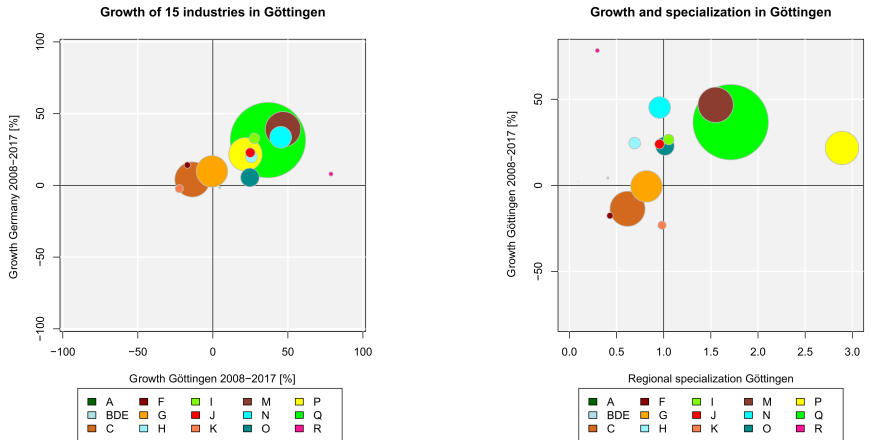
Both functions for commercial area prognosis (`gifpro()` and `gifpro.tbs()`) require vectors of employment data as well as the coefficients for resettlement etc. When forecasting commercial area using the trend-specific technique with `gifpro.tbs()`, the user needs time series data of previous industry-specific employment and has to specify a trend function type (linear, power, exponential or logistic) for each industry. The "best" function type may be examined visually by regarding the employment forecasting output (optional function argument `prog.plot = TRUE`) and the related R^2 values which is part of the invisible function output. Note that this function uses the `REAT` function `curvefit()`, which is a simple tool for bivariate regression, similar to the curve fitting functions in other spreadsheet or statistics software.

6.2.2 Application example 1: Analysis of regional growth in Göttingen

Referring to the example in Section 4.2.2, we perform a regional growth analysis for the German city Göttingen. We use the same dataset `Goettingen` as before, that contains industry-specific employment data for Göttingen and Germany from 2008 to 2017. We load our example data:

```
data(Goettingen)
```

In the first step, we want to examine the industry-specific growth in Göttingen visually. Using the function `portfolio()`, we plot a regional growth matrix with respect to the 15 industries (rows 2 to 16). We also set a plot title (argument `pmtitle`) and axis labels (arguments `pmx` and `pmy`, respectively) as well as industry-specific colors (argument `pcol`):



(a) Growth portfolio matrix for Göttingen (b) Growth-specialization matrix for Göttingen

Figure 6: Portfolio matrix analysis for 15 industries in Göttingen

```
portfolio (Goettingen$Goettingen2008[2:16],
Goettingen$Goettingen2017[2:16],
Goettingen$BRD2008[2:16], Goettingen$BRD2017[2:16],
psize = Goettingen$Goettingen2017[2:16], psize.factor = 15,
pmtitle = "Growth of 15 industries in Göttingen",
industry.names = Goettingen$WZ2008_Code[2:16],
pmx = "Growth Göttingen 2008-2017 [%]",
pmy = "Growth Germany 2008-2017 [%]",
pcol.border = "grey",
pcol = c("darkgreen", "powderblue", "chocolate", "darkred",
"orange", "cadetblue1", "chartreuse1", "red",  "coral",
"coral4", "cyan", "darkcyan", "yellow", "green", "deeppink"),
leg = TRUE, leg.x = -90)
```

Similarly, we plot a growth-specialization portfolio matrix using `locq.growth()` with the same options (colors etc.). On the y axis, we put the industry-specific regional growth which is stated by the function argument `y.axis = "r"` (if we would like to see the national growth instead, we had to set `y.axis = "n"`; for the quotient of regional and national growth, use `y.axis = "rn"`):

```
locq.growth (Goettingen$Goettingen2008[2:16],
Goettingen$Goettingen2017[2:16],
Goettingen$BRD2008[2:16], Goettingen$BRD2017[2:16],
psize = Goettingen$Goettingen2017[2:16], psize.factor = 15,
y.axis = "r", industry.names = Goettingen$WZ2008_Code[2:16],
pmtitle = "Growth and specialization in Göttingen",
pmx = "Regional specialization Göttingen",
pmy = "Growth Göttingen 2008-2017 [%]", pcol.border = "grey",
pcol = c("darkgreen", "powderblue", "chocolate", "darkred",
"orange", "cadetblue1", "chartreuse1", "red", "coral",
"coral4", "cyan", "darkcyan", "yellow", "green", "deeppink"),
leg = TRUE, leg.x = 0.1)
```

The resulting growth portfolio matrix is shown in Figure 6a, the growth-specialization portfolio in Figure 6b. The size of the points (or bubbles) is equal to the current industry-specific employment (e_{ij}) for 2017 (rows 2 to 16 of column `Goettingen2017` in the example data), normalized with respect to a maximum point size of 15 (argument `psize.factor = 15`). As we can see, the health sector (industry code Q, green bubble) has the highest absolute relevance, which can be attributed to the local university hospital (see Section 4.2.2). The axes in the growth portfolio are segmented at $x = 0$ and $y = 0$, respectively, which means a differentiation between positive and negative growth.

As we can see, most industries have grown from 2008 to 2017 in both the region and the whole economy (see quadrant I) with similar growth rates. There is one outlier: Industry R (arts, entertainment, and recreation) shows a regional growth of more than 75 percent, while the national growth is about 10 percent. Note that we see percentage growth rates from 2008 to 2017 here (if *average* growth rates are desired, use the function argument `time.periods`).

Looking at the growth-specialization portfolio, we can identify absolute relevance and growth rate as well as regional specialization of the industries (The colors and bubble sizes are equal to those in Figure 6a). In quadrant I, we find the industries which are overrepresented in Göttingen (specialization) and growing at this regional level. As expected in this university city and related to our results in Section 4.2.2, the "stars" in Göttingen are education (code P), health (code Q) and professional, scientific and technical services (code M).

While the portfolio matrix analysis tells us about the industry-specific growth, the shift-share analysis decomposes this growth into the national, industrial and regional components. In the first step, we perform a static shift-share analysis in the sense of Dunn Jr. (1960) for the same data as in the portfolio analysis by applying the function `shift()`:

```
shift(Goettingen$Goettingen2008[2:16], Goettingen$Goettingen2017[2:16],
Goettingen$BRD2008[2:16], Goettingen$BRD2017[2:16])
# rows 2-6: 15 industries
# columns Goettingen2008 and Goettingen2017:
# employment Goettingen 2008 and 2017, respectively
# columns BRD2008 and BRD2017:
# employment Germany 2008 and 2017, respectively
```

This is our (visible) output:

```
Shift-Share Analysis
Method: Dunn

Shift-share components
                Components
Growth (t1-t)   10411.0000
National share   9178.1916
Industrial mix   2204.8202
Regional share   -972.0118
Net total shift  1232.8084

Calculation for 15 industries
Regional employment at time t: 56872, at time t+1:
67283 (10411 / 18.30602 %)
National employment at time t: 27695398, at time t+1:
32164973 (4469575 / 16.13833 %)
```

In this cross-sectional analysis, we see that the overall employment in Göttingen increased by 10,411 persons from 2008 to 2017. However, a large share of this growth is due to the growth in the national economy ($n_{j_{t,t+y}} \approx 9,178$ employees), which is only a bit lower than Göttingen. The industrial mix component ($m_{j_{t,t+y}}$) shows that approximately 2,205 additional employees must be attributed to an overrepresentation of growing industries in Göttingen. The regional share is negative ($c_{j_{t,t+y}} \approx -972$), which indicates locational disadvantages. When interpreting the industrial mix also as a regional aspect (which seems plausible), we can look at the sum of the industrial mix and the regional share: The net total shift (t_{t+y}) is equal to 1,233 employees, representing the growth difference between the region and the whole economy.

We confirm our results using the Gerfin technique. We request it by setting the argument `shift.method` of the `shift()` function equal to `"Gerfin"`:

```
shift(Goettingen$Goettingen2008[2:16], Goettingen$Goettingen2017[2:16],
Goettingen$BRD2008[2:16], Goettingen$BRD2017[2:16],
shift.method = "Gerfin")
```

The output is:

```
Shift-Share Analysis
Method: Gerfin

Shift-share components
               Components
Industrial mix   1.0333810
Regional share   0.9857591
Net total shift  1.0186647

Calculation for 15 industries
Regional employment at time t: 56872, at time t+1:
67283 (10411 / 18.30602 %)
National employment at time t: 27695398, at time t+1:
32164973 (4469575 / 16.13833 %)
```

In the index method, there is no national share component (implicitly, it is equal to one), thus, we only take a look at the industrial mix and the regional share as well as the net total shift. The industrial mix component is above one ($n_{j_{t,t+y}} \approx 1.03$), showing a more advantageous sector structure in Göttingen compared to Germany. While the regional share in the Dunn-type shift-share analysis was negative, this component in the Gerfin analysis is slightly below one ($c_{j_{t,t+y}} \approx 0.99$), indicating locational disadvantages.

These traditional techniques only regard the overall growth with respect to cross-sectional data. To gain a deeper insight and take into account also seasonal effects, we perform a dynamic shift-share analysis in the sense of Barff, Knight (1988) which distinguishes between the 15 industries simultaneously. This can be done via the `REAT` function `shiftid()`, requiring data for the initial time period and at least for two following periods. In the `Goettingen` dataset, the rows 2 to 16 represent the industries and the columns represent the years (2008 to 2017). Data for the regarded region and the whole economy is arranged successively. We also use the industry codes in column `WZ2008_Code`. In this function, we have to define the start and end periods explicitly:

```
shiftid(Goettingen$Goettingen2008[2:16], Goettingen[2:16,3:12],
Goettingen$BRD2008[2:16], Goettingen[2:16,13:22],
time1 = 2008, time2 = 2017,
industry.names = Goettingen$WZ2008_Code[2:16])
# columns 3-12: employment in Göttingen 2009-2017
# columns 13-22: employment in Germany 2009-2017
```

The result is:

```
Dynamic Shift-Share Analysis
Method: Dunn

Shift-share components
                         A      BDE          C         F         G
Growth (t1-t)   -3.000000 29.00000 -1117.0000 -255.0000  -51.0000
National share   6.103502 -9.46377   254.5217  160.0638  561.7436
Regional share  -9.103502 38.46377 -1371.5217 -415.0638 -612.7436
Net total shift -9.103502 38.46377 -1371.5217 -415.0638 -612.7436
                        H         I         J           K        M
Growth (t1-t)   524.0000 470.00000 274.00000 -465.000000 2229.000
National share  368.2053 515.03493 286.32383    6.356612 1821.392
Regional share  155.7947 -45.03493 -12.32383 -471.356612  407.608
Net total shift 155.7947 -45.03493 -12.32383 -471.356612  407.608
                        N        O         P        Q         R
Growth (t1-t)   1178.0000 268.0000 1272.0000 4211.000 363.00000
National share   977.9869 167.9118 1138.5383 3556.692  47.50353
Regional share   200.0131 100.0882  133.4617  654.308 315.49647
Net total shift  200.0131 100.0882  133.4617  654.308 315.49647
```

```
Calculation for 15 industries
Regional employment at time t: 56872, at time t+1:
67283 (10411 / 18.30602 %)
National employment at time t: 27695398, at time t+1:
32164973 (4469575 / 16.13833 %)
```

The visible output is a `matrix` containing one row for each component (the number of components depends on the selected shift-share method, here: Dunn) and I columns (one for each industry). As we calculate industry-specific components, there is no industrial mix effect, which means that the calculations are on the level of single industries. Again, we detect large absolute growth for industries P (education) and Q (health) (see Table 9). Interestingly, this growth can be mainly attributed to effects in the whole economy. The corresponding regional shares are small but positive, showing locational advantages with respect to these industries in Göttingen.

The logic of shift-share analysis can also be regarded in two other examples: If industry C (manufacturing) *had* developed as in the national trend, the absolute growth in Göttingen *would* be equal to 255 employees. In fact, there was a decline of 1,117 employees, resulting in a negative regional share of -1,372 employees, indicating locational disadvantages with respect to the manufacturing sector. The opposite is true for the industries with code BDE (including electricity, gas, water supply, etc.): The absolute growth of 29 employees *would* not have occurred if this sector *had* developed as in the whole economy (negative national share equal to -9 employees). The residuum (regional share) is equal to 38 employees, indicating a trend contrary to the national.

6.2.3 Application example 2: Commercial area prognosis for Göttingen

Using the same data, we now perform a commercial area prognosis for Göttingen. We load our data:

```
data(Goettingen)
```

When using the GIFPRO-based commercial area prognosis techniques, several parameters have to be defined (employment shares in commercial areas a_i, resettlement rate sq_{ij}, relocation rate rq_{ij} and areal index ai_{ij}; a reutilization rate ru_{ij} is optional, thus, we ignore the reutilization of commercial area in this example). These parameters have to be defined for each industry. In our example, we use the employment shares as well as the resettlement and relocation rates from Deutsches Institut für Urbanistik GmbH, Spath + Nagel (GbR) (2010). Note that some sectors are, per definition, not located within commercial areas (e.g. agriculture), resulting in an employment share of $a_i = 0$. As we want to reuse the sets of parameters, we save them as single `numeric vectors`:

```
ca_share <- c(0, 0, 100, 90, 70, 100, 10, 20, 20, 20, 20, 0, 0, 0, 0)
# industry-specific shares of employees in commercial areas
sq_quote <- c(0.77, 0.77, 0.15, 0.15, 0.77, 0.15, 0.77, 0.77,
0.77, 0.77, 0.77, 0.77, 0.77, 0.77, 0.77)
# industry-specific resettlement quote
rq_quote <- rep(0.7, 15)
# industry-specific relocation quote (0.7 for each of the 15 industries)
area_index <- c(0, 0, 200, 75, 250, 250, 50, 100, 100, 100, 100,
50, 50, 50, 50)
# industry-specific area index (sqm commercial area per employee)
```

Now, we compute the traditional commercial area prognosis using the `gifpro()` function and the `Goettingen` data as well as the parameters defined above. We forecast the commercial area for five years (`tinterval = 5`). Our base is 2017 (`time.base = 2017`), as this is the last year empirical data is available for. We save the (invisible) output in the `list` object `gifpro_goettingen`:

```
gifpro_goettingen <- gifpro (e_ij = Goettingen$Goettingen2017[2:16],
a_i = ca_share, sq_ij = sq_quote, rq_ij = rq_quote, tinterval = 5,
ai_ij = area_index, time.base = 2017,
industry.names = Goettingen$WZ2008_Code[2:16], output = "full")
```

As we have set `output = "full"`, the visible function output contains overall as well as annual values:

```
GIFPRO
Method: GIFPRO

Employment and commercial area changes (allover)
        Employment Commercial Area
Sum      1113.8785       212981.94
Average   222.7757        42596.39

Employment and commercial area changes (per time unit)
     Employment CommercialArea
2018   222.7757       42596.39
2019   222.7757       42596.39
2020   222.7757       42596.39
2021   222.7757       42596.39
2022   222.7757       42596.39

Calculation for 15 industries
```

In all 15 industries, 1,114 new employees are predicted for the year 2022, resulting in 212,928 square meters required for new commercial area. As the employment prognosis is not based on (nonlinear) trend regression but on constant growth, the absolute employment growth and the required commercial area are equal in each year (223 employees and 42,596 sqm, respectively).

The object `gifpro_goettingen` contains a `list` called `components` containing the single components of prognosis as well as the results already shown in the visible output (`results`). To understand the GIFPRO technique and the related `REAT` function, we take a look at the single components:

```
gifpro_goettingen$components

$resettlement
        2018     2019     2020     2021     2022
A    0.00000  0.00000  0.00000  0.00000  0.00000
BDE  0.00000  0.00000  0.00000  0.00000  0.00000
C   11.81100 11.81100 11.81100 11.81100 11.81100
F    1.80090  1.80090  1.80090  1.80090  1.80090
G   38.00489 38.00489 38.00489 38.00489 38.00489
H    3.72150  3.72150  3.72150  3.72150  3.72150
I    1.73327  1.73327  1.73327  1.73327  1.73327
J    3.12928  3.12928  3.12928  3.12928  3.12928
K    2.67806  2.67806  2.67806  2.67806  2.67806
M   12.18910 12.18910 12.18910 12.18910 12.18910
N    7.50750  7.50750  7.50750  7.50750  7.50750
O    0.00000  0.00000  0.00000  0.00000  0.00000
P    0.00000  0.00000  0.00000  0.00000  0.00000
Q    0.00000  0.00000  0.00000  0.00000  0.00000
R    0.00000  0.00000  0.00000  0.00000  0.00000

$relocation
       2018    2019    2020    2021    2022
A    0.0000  0.0000  0.0000  0.0000  0.0000
BDE  0.0000  0.0000  0.0000  0.0000  0.0000
C   55.1180 55.1180 55.1180 55.1180 55.1180
F    8.4042  8.4042  8.4042  8.4042  8.4042
G   34.5499 34.5499 34.5499 34.5499 34.5499
H   17.3670 17.3670 17.3670 17.3670 17.3670
I    1.5757  1.5757  1.5757  1.5757  1.5757
J    2.8448  2.8448  2.8448  2.8448  2.8448
K    2.4346  2.4346  2.4346  2.4346  2.4346
```

```
M   11.0810 11.0810 11.0810 11.0810 11.0810
N    6.8250  6.8250  6.8250  6.8250  6.8250
O    0.0000  0.0000  0.0000  0.0000  0.0000
P    0.0000  0.0000  0.0000  0.0000  0.0000
Q    0.0000  0.0000  0.0000  0.0000  0.0000
R    0.0000  0.0000  0.0000  0.0000  0.0000

$reuse
    2018 2019 2020 2021 2022
A      0    0    0    0    0
BDE    0    0    0    0    0
C      0    0    0    0    0
F      0    0    0    0    0
G      0    0    0    0    0
H      0    0    0    0    0
I      0    0    0    0    0
J      0    0    0    0    0
K      0    0    0    0    0
M      0    0    0    0    0
N      0    0    0    0    0
O      0    0    0    0    0
P      0    0    0    0    0
Q      0    0    0    0    0
R      0    0    0    0    0

$employment
        2018     2019     2020     2021     2022
A    0.00000  0.00000  0.00000  0.00000  0.00000
BDE  0.00000  0.00000  0.00000  0.00000  0.00000
C   66.92900 66.92900 66.92900 66.92900 66.92900
F   10.20510 10.20510 10.20510 10.20510 10.20510
G   72.55479 72.55479 72.55479 72.55479 72.55479
H   21.08850 21.08850 21.08850 21.08850 21.08850
I    3.30897  3.30897  3.30897  3.30897  3.30897
J    5.97408  5.97408  5.97408  5.97408  5.97408
K    5.11266  5.11266  5.11266  5.11266  5.11266
M   23.27010 23.27010 23.27010 23.27010 23.27010
N   14.33250 14.33250 14.33250 14.33250 14.33250
O    0.00000  0.00000  0.00000  0.00000  0.00000
P    0.00000  0.00000  0.00000  0.00000  0.00000
Q    0.00000  0.00000  0.00000  0.00000  0.00000
R    0.00000  0.00000  0.00000  0.00000  0.00000
```

As we defined some industries as not relevant for commercial areas ($a_i = 0$), they do not contribute any employees neither resettled nor relocated (such as A - agriculture, B - mining and quarrying or R - arts, entertainment, and recreation). We see that e.g. in the manufacturing sector (code C), there is an annual increase of about 12 employees attributed to resettlement and 55 employees related to relocation each year (see row 3 in `resettlement` and `relocation`, respectively). As we ignored the reutilization of commercial area, the `matrix` containing the commercial area-relevant employment related to reutilization (`reuse`) contains only zeros. The sum of all three components is stored in the fourth `matrix`, `employment`. There is an annual increase of nearly 67 employees in the manufacturing sector. The contents of the `results list` is the same as shown in the visible output.

In the next step, we apply the trend-based commercial area prognosis (TBS-GIFPRO) to the `Goettingen` data. In the `gifpro.tbs()` function, we use the employment data from 2008 to 2017 (columns 3 to 12), and assume an exponential function for employment prognosis (function argument `prog.func`, repeating the argument `"exp"` for each industry). The employment prognosis is plotted (`prog.plot = TRUE`), showing all 15 plots in one (`plot.single = FALSE`):

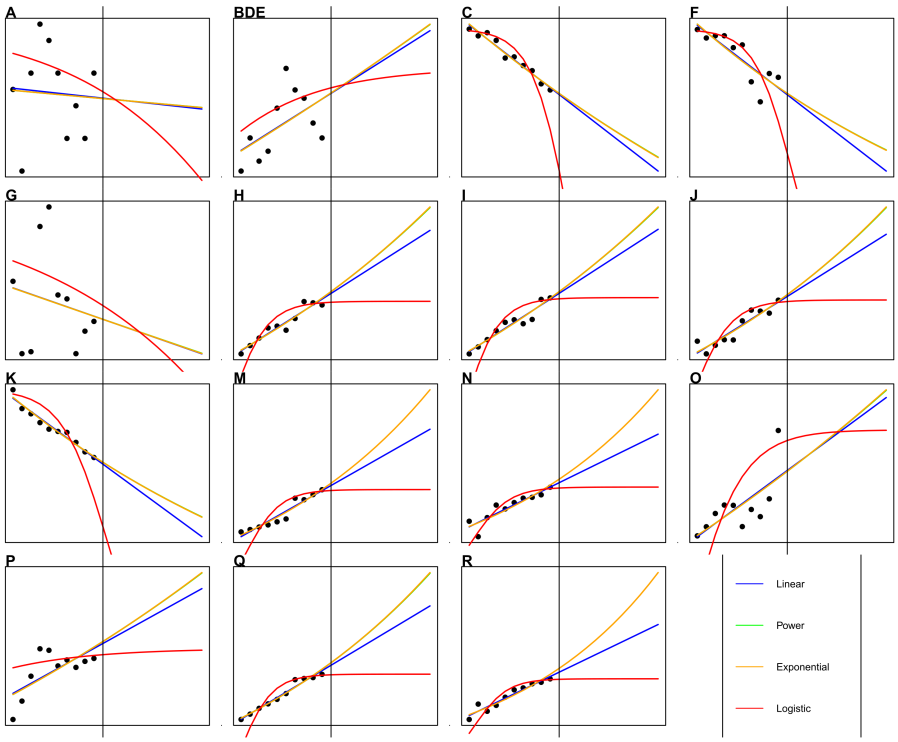


Figure 7: Employment prognosis for 15 industries in Goettingen (TBS-GIFPRO)

```
gifpro.tbs (e_ij = Goettingen[2:16,3:12],
a_i = ca_share, sq_ij = sq_quote, rq_ij = rq_quote, tinterval = 5,
prog.func = rep("exp", nrow(Goettingen[2:16,3:12])),
ai_ij = area_index, time.base = 2008,
industry.names = Goettingen$WZ2008_Code[2:16],
prog.plot = TRUE, plot.single = FALSE, output = "full")
```

The visible function output is similar to the output above:

```
GIFPRO
Method: TBS-GIFPRO

Employment and commercial area changes (allover)
        Employment CommercialArea
Sum      1139.6592      216012.46
Average   227.9318       43202.49

Employment and commercial area changes (per time unit)
     Employment CommercialArea
2018   224.9565       42945.53
2019   226.2904       43054.76
2020   227.7755       43182.97
2021   229.4169       43330.70
2022   231.2199       43498.50

Calculation for 10 industries
```

The resulting plot containing the employment forecasting functions is shown in Figure 7. The black vertical lines divide the plots into the esimation segment (2008 to 2017) and the prognosis segment (2018 to 2022). Four function types are supplied: linear (blue), power (green), exponential (yellow) and logistic (red). Note that a linear trend seems unrealistic as it implies continuous growth and may result in negative employment if the slope is negative. At this point, we should normally discuss and find the "best"

forecasting model for each industry and rerun our analysis a few times. In our example, we skip this step and just take a look at the prognosis functions: In most cases, an exponential growth (or decline) seems to be an appropriate approximation. The power functions (green lines) are nearly invisible as their data fit is nearly the same as that of the exponential functions. Thus, we could choose them instead. In our case, the exponential function seems sufficient.

As expected, a nonlinear industry growth results in a nonlinear overall employment growth and, consequently, the commercial area-relevant employment also grows in a nonlinear way. As we can see from the `gifpro.tbs()` output, employment increases by about 228 employees per year on average and by about 1,140 employees over the five years regarded (2018 to 2022). The annual commercial area required ranges from 42,946 sqm (2018) to 43,499 sqm (2022), all in all 216,012 sqm up to 2022. In our case, the estimated commercial area exceeds the prognosis derived from the simple GIFPRO analysis, which can be attributed to the positive differences between the exponential prognosis and a linear prognosis (see Figure 7). We skip the inspections of the components, which could be addressed by saving the results in an object (`list`), as we did in the first GIFPRO example.

7 Final remarks

This paper has shown how `R` and specifically the package `REAT` can be used for regional economic analysis. It should be noted that this package aims at width with respect to the treated analysis subjects rather than depth. The subsections provide the basic analysis methods regarded as most important from the package developer's point of view (with respect to usage in current papers and discussion in current textbooks as well as application in own research projects), while there are several other approaches as well as extensions of the basic methods. A more detailed survey of the common methods can be found in the cited literature, especially in review articles (e.g. Nakamura, Morrison Paul 2009; Portnov, Felsenstein 2010) and textbooks (e.g. Farhauer, Kröll 2014).

Finally, we have to keep in mind that this package (like nearly any other free software) was developed in a non-commercial context (and published under the GNU General Public License). All functions have been tested several times using various real data and single functions have already been used in a few research projects. However, there is no warranty that all functions always work perfectly. Like nearly any other `R` package, `REAT` is continuously refined, which means extending functions as well as correcting errors. This requires attentive usage and, of course, constructive feedback from the package users. It can be easily transmitted using the contact information on the CRAN package website.

References

Albacete X, Olaru D, Paül V, Biermann S (2017) Measuring the accessibility of public transport: A critical comparison between methods in Helsinki. *Applied Spatial Analysis and Policy* 10[2]: 161–188. CrossRef.

Allington NF, McCombie J (2007) Economic Growth and Beta-Convergence in the East European Transition Economies. In: Arestis P, Baddeley M, McCombie J (eds), *Economic Growth*. Edward Elgar publishing, Cheltenham, 200–222

Arcelus FJ (1984) An extension of shift-share analysis. *Growth and Change* 15[1]: 3–8. CrossRef.

Bai CE, Tao Z, Tong YS (2008) Bureaucratic integration and regional specialization in China. *China Economic Review* 19[2]: 308–319. CrossRef.

Baker P, von Kirchbach F, Mimouni M, Pasteels JM (2002) Analytical Tools for Enhancing the Participation of Developing Countries in the Multilateral Trading System in the Context of the Doha Development Agenda. *Aussenwirtschaft* 57[3]: 343–372. https://EconPapers.repec.org/RePEc:usg:auswrt:2002:57:03:343-372

Balassa B (1965) Trade Liberalisation and "Revealed" Comparative Advantage. *The Manchester School* 33[2]: 99–123. CrossRef.

Barff RA, Knight PL (1988) Dynamic shift-share analysis. *Growth and Change* 19[2]: 1–10. CrossRef.

Barro RJ, Sala-i Martin X (2004) *Economic Growth* (2nd ed.). MIT Press

Bonny HW, Kahnert R (2005) Zur Ermittlung des Gewerbeflächenbedarfs. *Raumforschung und Raumordnung* 63[3]: 232–240

Capello R, Nijkamp P (2009) Introduction: Regional growth and development theories in the twenty-first century - recent theoretical advances and future challenges. In: Capello R, Nijkamp P (eds), *Handbook of Regional Growth and Development Theories*. 1–18

Casler SD (1989) A Theoretical Context for Shift and Share Analysis. *Regional Studies* 23[1]: 43–48. CrossRef.

Ceapraz IL (2008) The Concepts of Specialisation and Spatial Concentration and the Process of Economic Integration: Theoretical Relevance and Statistical Measures. The Case of Romania's Regions. *Romanian Journal of Regional Science* 2[1]: 68–93

Charles-Coll JA (2011) Unterstanding Income Equality: Concept, Causes and Management. *International Journal of Economics and Management Science* 1[3]: 17–28

CIMA Projekt + Entwicklung GmbH, NIW Niedersächsisches Institut für Wirtschaftsforschung, NORD/LB Regionalwirtschaft, Planquadrat Dortmund GbR (2011) Gewerbeflächenkonzeption für die Metropolregion Hamburg (GEFEK). Research report

Cracau D, Durán Lima JE (2016) On the Normalized Herfindahl-Hirschman Index: A Technical Note. *International Journal on Food System Dynamics* 7[4]: 382–386

Damgaard C, Weiner J (2000) Describing inequality in plant size or fecundity. *Ecology* 81[4]: 1139–1142. CrossRef.

Dapena AD, Fernández Vázquez E, Rubiera Morollón F (2016) The role of spatial scale in regional convergence: the effect of MAUP in the estimation of β-convergence equations. *The Annals of Regional Science* 56[2]: 473–489. CrossRef.

Dauth W, Fuchs M, Otto A (2015) Standortmuster in Westdeutschland: Nur wenige Branchen sind räumlich stark konzentriert. IAB Kurzbericht 16/2015, Institut für Arbeitsmarkt- und Berufsforschung. http://doku.iab.de/kurzber/2015/kb1615.pdf

Dauth W, Fuchs M, Otto A (2018) Long-run processes of geographical concentration and dispersion: Evidence from Germany. *Papers in Regional Science* 97[3]: 569–593. CrossRef.

Deutsches Institut für Urbanistik GmbH, Spath + Nagel (GbR) (2010) Stadtentwicklungskonzept Gewerbe für die Landeshauptstadt Potsdam. Research report, Landeshauptstadt Potsdam. https://www.potsdam.de/sites/default/files/documents/STEK_Gewerbe_Langfassung_2010.pdf

Dinc M (2015) *Introduction to Regional Economic Development. Major Theories and Basic Analytical Tools*. Elgar

Dixon R, Freebairn J (2009) Trends in Regional Specialisation in Australia. *Australasian Journal of Regional Studies* 15[3]: 281–296

Doran J, Jordan D (2013) Decomposing European NUTS2 regional inequality from 1980 to 2009: National and European policy implications. *Journal of Economic Studies* 40[1]: 22–38. CrossRef.

Dunn Jr. ES (1960) A Statistical and Analytical Technique for Regional Analysis. *Papers in Regional Science* 6[1]: 97–112. CrossRef.

Duranton G, Puga D (2000) Diversity and Specialisation in Cities: Why, Where and When Does it Matter? *Urban Studies* 37[3]: 533–555. CrossRef.

Ellison G, Glaeser E (1997) Geographic Concentration in U.S. Manufacturing Industries: A Dartboard Approach. *Journal of Political Economy* 105[5]: 889–927

Espa G, Arbia G, Giuliani D (2010) Measuring industrial agglomeration with inhomogeneous K-function: the case of ICT firms in Milan (Italy). Department of Economics Working Papers 1014, Department of Economics, University of Trento, Italia

Esteban-Marquillas JM (1972) I. A reinterpretation of shift-share analysis. *Regional and Urban Economics* 2[3]: 249 – 255. CrossRef.

Farhauer O, Kröll A (2014) *Standorttheorien. Regional- und Standortökonomik in Theorie und Praxis* (2nd ed.). Springer, Heidelberg

Fujita M, Krugman P, Venables A (2001) *The Spatial Economy: Cities, Regions, and International Trade* (1st ed.), Volume 1. The MIT Press

Fülöp G, Kopetsch T, Schöpe P (2011) Catchment areas of medical practices and the role played by geographical distance in the patient's choice of doctor. *The Annals of Regional Science* 46[3]: 691–706. CrossRef.

Furceri D (2005) Beta and sigma convergence: A mathematical relation of causality. *Economics Letters* 89[2]: 212–215. CrossRef.

Gerfin H (1964) Gesamtwirtschaftliches Wachstum und regionale Entwicklung. *Kyklos* 17[4]: 565–593. CrossRef.

Gini C (1912) *Variabilità e Mutuabilità. Contributo allo Studio delle Distribuzioni e delle Relazioni Statistiche*. Cuppini

Gluschenko K (2018) Measuring regional inequality: to weight or not to weight? *Spatial Economic Analysis* 13[1]: 36–59. CrossRef.

Goecke H, Hüther M (2016) Regional Convergence in Europe. *Intereconomics* 51[3]: 165–171. CrossRef.

Goschin Z, Constantin D, Roman M, Ileanu B (2009) Regional specialization and geographic concentration of industries in Romania. *South-Eastern Europe Journal of Economics* 1[1]: 99–113. https://ojs.lib.uom.gr/index.php/seeje/article/view/5536

Haas A, Südekum J (2005) Spezialisierung und Branchenkonzentration in Deutschland: Regionalanalyse. IAB-Kurzbericht 1/2005. http://hdl.handle.net/10419/158181

Habánik J, Hošták P, Kútik J (2013) Economic and social disparity development within regional development of the Slovak Republic. *Economics and Management* 18[3]: 457–464. CrossRef.

Hansen WG (1959) How Accessibility Shapes Land Use. *Journal of the American Institute of Planners* 25[2]: 73–76. CrossRef.

Harris CD (1954) The Market as a Factor in the Localization of Industry in the United States. *Annals of the Association of American Geographers* 44[4]: 315–348

Haynes KE, Parajuli J (2014) Shift-share analysis: decomposition of spatially integrated systems. In: *Handbook of Research Methods and Applications in Spatially Integrated Social Science*. Elgar, 315–344. CrossRef.

Heinemann M (2008) Messung und Darstellung von Ungleichheit. Working Paper Series in Economics 108, University of Lüneburg, Institute of Economics. https://EconPapers.repec.org/RePEc:lue:wpaper:108

Henderson BD (1973) The Experience Curve - Reviewed. IV. The Growth Share Matrix or The Product Portfolio. Reprint 135. https://www.bcg.com/documents/file13904.pdf

Herfindahl OC (1950) *Concentration in the U.S. Steel Industry*. Colombia University Press

Hirschman AO (1945) *National Power and the Structure of Foreign Trade*. Publications of the Bureau of Business and Economic Research. University of California Press

Hoen AR, Oosterhaven J (2006) On the measure of comparative advantage. *The Annals of Regional Science* 40[3]: 677–691. CrossRef.

Hoffmann J, Hirsch S, Simons J (2017) Identification of spatial agglomerations in the German food processing industry. *Papers in Regional Science* 96[1]: 139–162. CrossRef.

Hoover EM (1936) The Measurement of Industrial Localization. *The Review of Economics and Statistics* 18[4]: 162–171

Howard D (2007) A regional economic performance matrix – an aid to regional economic policy development. *Journal of Economic and Social Policy* 11[2]: article 4. https://EconPapers.repec.org/RePEc:usg:auswrt:2002:57:03:343-372

Howard E, Newman C, Tarp F (2016) Measuring industry coagglomeration and identifying the driving forces. *Journal of Economic Geography* 16[5]: 1055–1078

Huang Y, Leung Y (2009) Measuring Regional Inequality: A Comparison of Coefficient of Variation and Hoover Concentration Index. *The Open Geography Journal* 2[1]: 25–34. CrossRef.

Jiang L, Guan M, Tian J (2007) On Chinese Regional Specialization and Industry Concentration. In: *2007 International Conference on Machine Learning and Cybernetics*, Volume 6, 3396–3400

Kabacoff RI (2017) Quick-R: Data Types. Manual. https://www.statmethods.net/input/datatypes.html

Kassenärztliche Bundesvereinigung (2013) Die neue Bedarfsplanung. Grundlagen, Instrumente und regionale Möglichkeiten. Brochure. https://www.kbv.de/media/sp/Instrumente_Bedarfsplanung_Broschuere.pdf

Kim S (1995) Expansion of Markets and the Geographic Distribution of Economic Activities: The Trends in U. S. Regional Manufacturing Structure, 1860–1987. *The Quarterly Journal of Economics* 110[4]: 881–908. CrossRef.

Kiskowski MA, Hancock JF, Kenworthy A (2009) On the Use of Ripley's K-function and its Derivatives to Analyze Domain Skriderize. *Biophysical Journal* 97[4]: 1095–1103. CrossRef.

Kohn W, Öztürk R (2013) *Statistik für Ökonomen. Datenanalyse mit R und SPSS* (2nd ed.). Springer Gabler

Krider R, Putler DS (2013) Which Birds of a Feather Flock Together? Clustering and Avoidance Patterns of Similar Retail Outlets. *Geographical Analysis* 45[2]: 123–149. CrossRef.

Krugman P (1979) Increasing returns, monopolistic competition, and international trade. *Journal of International Economics* 9[4]: 469–479. CrossRef.

Krugman P (1991) *Geography and trade.* MIT Press

Larsson JP, Öner Ö (2014) Location and co-location in retail: a probabilistic approach using geo-coded data for metropolitan retail markets. *The Annals of Regional Science* 52[2]: 385–408. CrossRef.

Lehocký F, Rusnák J (2016) Regional specialization and geographic concentration: experiences from Slovak industry. *Miscellanea Geographica – Regional Studies on Development* 20[3]: 5–13. https://www.degruyter.com/downloadpdf/j/mgrsd.2016.20.issue-3/mgrsd-2016-0011/mgrsd-2016-0011.pdf

Lessmann C (2005) Regionale Disparitäten in Deutschland und ausgesuchten OECD-Staaten im Vergleich. *ifo Dresden berichtet* 3/2005: 25–33

Lessmann C (2014) Spatial inequality and development - Is there an inverted-U relationship? *Journal of Development Economics* 106: 35–51. CrossRef.

Lessmann C (2016) Regional inequality and internal conflict. *German Economic Review* 17[2]: 157–191. CrossRef.

Lessmann C, Seidel A (2017) Regional inequality, convergence, and its determinants – a view from outer space. *European Economic Review* 92: 110–132. CrossRef.

Litzenberger T, Sternberg R (2006) Der Clusterindex – eine Methodik zur Identifizierung regionaler Cluster am Beispiel deutscher Industriebranchen. *Geographische Zeitschrift* 94[2]: 209–224

Longley PA, Goodchild MF, Maguire DJ, Rhind DW (2005) *Geographical Information Systems and Science* (2nd ed.). Wiley

Lorenz MO (1905) Methods of measuring the concentration of wealth. *Publications of the American Statistical Association* 9[70]: 209–219. CrossRef.

Martin C (2015) Kreative Klasse 2015. Kreativität als entscheidender Faktor für wirtschaftlichen Erfolg: Entwicklungen und Ausprägungen in Deutschland. Research report. https://www.kreativ-sta.de/wp-content/uploads/2017/10/agiplan_Kreative_Klasse_2015_Studie.pdf

Midelfart-Knarvik K, Overman H, Redding S, Venables A (2000) The Location of European Industry. *European Economy - Economic Papers* 142

Moga LM, Constantin DL (2011) Specialization and Geographic Concentration of the Economic Activities in the Romanian Regions. *Journal of Applied Quantitative Methods* 6[2]: 12–21. https://pdfs.semanticscholar.org/aa9d/365d6a8ef4c3585595c8ba03fe373ab02010.pdf

Mulligan GF (2006) Logistic Population Growth in the World's Largest Cities. *Geographical Analysis* 38[4]: 344–370. CrossRef.

Mussini M (2017) Decomposing Changes in Inequality and Welfare Between EU Regions: The Roles of Population Change, Re-Ranking and Income Growth. *Social Indicators Research* 130[2]: 455–478. CrossRef.

Myrdal G (1957) *Economic theory and under-developed regions*. G. Duckworth

Nakamura R, Morrison Paul C (2009) Measuring agglomeration. In: Capello R, Nijkamp P (eds), *Handbook of Regional Growth and Development Theories*. Elgar, 305–328

Nischwitz G, Böhme R, Fortmann F (2017) Kommunale Wirtschaftsförderung in Bremen: Handlungsrahmen, Programme und Wirkungen. Schriftenreihe Institut Arbeit und Wirtschaft 23/2017. http://hdl.handle.net/10419/172756

O'Donoghue D, Gleave B (2004) A Note on Methods for Measuring Industrial Agglomeration. *Regional Studies* 38[4]: 419–427. CrossRef.

OECD (2019) OECD Territorial Reviews. Website. https://www.oecd-ilibrary.org/fr/urban-rural-and-regional-development/oecd-territorial-reviews_19900759

Palan N (2017) Konzentrations- und Ungleichheitsindizes: ein methodischer Überblick sowie ein empirischer Vergleich anhand der Textilindustrie. *Zeitschrift für Wirtschaftsgeographie* 61[3-4]: 135–155. CrossRef.

Peña Carrera L (2002) Tracing accessibility over time: two swiss case studies. Technical report. http://hdl.handle.net/2099.1/6327

Petrakos G, Psycharis Y (2016) The spatial aspects of economic crisis in Greece. *Cambridge Journal of Regions, Economy and Society* 9[1]: 137–152. CrossRef.

Planungsgruppe MWM (2009) Flächennutzungsplanung Gemeinde Wachtberg - Fachbeitrag Arbeiten. Report. http://www.wachtberg.de/imperia/md/content/cms127/gemeindeentwicklung/fnp-fb-arbeiten-24-02-2009.pdf

Pooler J (1987) Measuring geographical accessibility: a review of current approaches and problems in the use of population potentials. *Geoforum* 18[3]: 269 – 289. CrossRef.

Porter ME (1990) *The Competitive Advantage of Nations*. Free Press

Portnov BA, Felsenstein D (2005) Measures of Regional Inequality for Small Countries. In: Felsenstein D, Portnov B (eds), *Regional Disparities in Small Countries*. 47–62. CrossRef.

Portnov BA, Felsenstein D (2010) On the suitability of income inequality measures for regional analysis: Some evidence from simulation analysis and bootstrapping tests. *Socio-Economic Planning Sciences* 44[4]: 212–219. CrossRef.

Puente S (2017) Regional convergence in Spain: 1980-2015. Research report. Economic Bulletin 3/2017, Banco de Espana

R Core Team (2018a) R: A Language and Environment for Statistical Computing. Software, Vienna, Austria. https://www.R-project.org/

R Core Team (2018b) The R Manuals. Manual. https://cran.r-project.org/manuals.html

Reggiani A, Bucci P, Russo G (2011) Accessibility and Impedance Forms: Empirical Applications to the German Commuting Network. *International Regional Science Review* 34[2]: 230–252. CrossRef.

Ricardo D (1821) *On the Principles of Political Economy and Taxation* (3rd ed.). McMaster University Archive for the History of Economic Thought

Ripley BD (1976) The second-order analysis of stationary point processes. *Journal of Applied Probability* 13[2]: 255–266. CrossRef.

RStudio Team (2016) RStudio: Integrated Development Environment for R. Software, RStudio, Inc., Boston, MA. http://www.rstudio.com/

Schmidt H (1997) *Konvergenz wachsender Volkswirtschaften. Theoretische und empirische Konzepte sowie eine Analyse der Produktivitätsniveaus westdeutscher Regionen*, Volume 152 of *Wirtschaftswissenschaftliche Beiträge*. Springer

Schönebeck C (1996) *Wirtschaftsstruktur und Regionalentwicklung : theoretische und empirische Befunde für die Bundesrepublik Deutschland*, Volume 75 of *Dortmunder Beiträge zur Raumplanung Blaue Reihe*. IRPUD

Schätzl L (2000) *Wirtschaftsgeographie 2: Empirie* (3rd ed.). Schöningh

Smith TE (2016) Notebook on Spatial Data Analysis. Technical report. http://www.seas.upenn.edu/~ese502/#notebook

Spiekermann K, Wegener M (2008) Modelle in der Raumplanung I: 4. Input-Output-Modelle. Presentation, Lecture "Modelle in der Raumplanung" WS 2008/2009. http://www.spiekermann-wegener.de/mir/pdf/MIR1_4_111108.pdf

Stark KD, Velsinger P, Bauer M, Bonny HW, Kricke J, Schwetlick D, Striedel HD (1981) *Flächenbedarfsberechnung für Gewerbe- und Industrieansiedlungsbereiche: GIFPRO*. Number 4.029 in Schriftenreihe Landes- und Stadtentwicklungsforschung des Landes Nordrhein-Westfalen. ILS, Dortmund

Statistisches Bundesamt (2008) German Classification of Economic Activities, Edition 2008. Dataset (XLS). https://www.destatis.de/DE/Methoden/Klassifikationen/GueterWirtschaftklassifikationen/klassifikationWZ08englisch.xls

Störmann W (2009) *Regionalökonomik. Theorie und Praxis*. Oldenbourg, Munich

Taylor JK, Cihon C (2004) *Statistical Techniques for Data Analysis* (2nd ed.). Taylor and Francis

Theil H (1967) *Economics and information theory*. North-Holland

Tian Z (2013) Measuring agglomeration using the standardized location quotient with a bootstrap method. *Journal of Regional Analysis and Policy* 43[2]: 186–197

Vallée D, Witte A, Brandt T, Bischof T (2012) Bedarfsberechnung für die Darstellung von Allgemeinen Siedlungsbereichen (ASB) und Gewerbe- und Industrieansiedlungsbereichen (GIB) in Regionalplänen. Research report, Staatskanzlei des Landes Nordrhein-Westfalen. https://www.wirtschaft.nrw/sites/default/files/asset/document/lep_nrw_flaechenbedarf_endbericht_endfassung_04122012.pdf

Vogiatzoglou K (2006) Increasing agglomeration or dispersion? Industrial specialization and geographic concentration in NAFTA. *Journal of Economic Integration* 21[2]: 379–396

von Neumann J, Kent RH, Bellinson HR, Hart BI (1941) The Mean Square Successive Difference. *The Annals of Mathematical Statistics* 12[2]: 153–162. CrossRef.

Weddige-Haaf K, Kool C (2017) Determinants of regional growth and convergence in Germany. Discussion paper. Discussion Paper Series 17-12, Utrecht University School of Economics

Wieland T (2019) REAT: Regional Economic Analysis Toolbox. R package version 3.0.1. Software. https://CRAN.R-project.org/package=REAT

Wieland T, Dittrich C (2016) Bestands- und Erreichbarkeitsanalyse regionaler Gesundheitseinrichtungen in der Gesundheitsregion Göttingen. Research report, Georg-August-Universität Göttingen, Geographisches Institut, Abteilung Humangeographie. http://webdoc.sub.gwdg.de/pub/mon/2016/3-wieland.pdf

Wieland T, Fuchs H (2018) Regionalökonomische Disparitäten im Spiegel von Raumtypisierungen. Ein Konzept zur Identifikation strukturell benachteiligter Gebiete in Südtirol (Italien). *Standort - Zeitschrift für Angewandte Geographie* 42[3]: 152–163. CrossRef.

Williamson JG (1965) Regional Inequality and the Process of National Development: A Description of the Patterns. *Economic Development and Cultural Change* 13[4]: 1–84

Yamamura S, Goto H (2018) Location patterns and determinants of knowledge-intensive industries in the Tokyo Metropolitan Area. *Japan Architectural Review* 1[4]: 443–456. CrossRef.

Young AT, Higgins MJ, Levy D (2008) Sigma Convergence versus Beta Convergence: Evidence from U.S. County-Level Data. *Journal of Money, Credit and Banking* 40[5]: 1083–1093. CrossRef.

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